[illegible]

by Roger B. Myerson

March 2010, revised Feb 2011

http://home.uchicago.edu/~rmyerson/research/bankers.pdf

Abstract:

long-term assets. In the Bernanke-Gertler and Suarez-Sussman models, the dynamic state variable is the aggregate wealth of entrepreneurs, who are subject to moral hazard in the second period of their two-period careers, but our model here shifts the focus to moral hazard of financial intermediaries whose careers can span any given number of periods. As in other standard models of financial intermediation (Diamond, 1984; Holmstrom and Tirole, 1997; Philippon, 2008), the problem of moral hazard in financial intermediation is derived here from a more basic problem of moral hazard in entrepreneurship, but financial intermediaries here are distinguished by their long-term relationships with investors.

Several other recent papers have also offered theoretical models to show how macroeconomic instability can be derived from incentive constraints in microeconomic transactions. Like this paper, Sussman and Suarez (1997, 2007) and Li and Wang (2010) have dynamic models in which macroeconomic fluctuations are driven by purely moral hazard in one sector, with no exogenous shocks. Closely related models that involve adverse selection have been developed by Azariadis and Smith (1998), Reichlin and Siconolfi (2004), Martin (2008), and Figueroa and Leukhina (2010). Other important credit-cycle models of Kiyotaki and Moore (1997), He and Krishnamurthy (2008), and Brunnermeier and Sannikov (2009) have analyzed how the prices of long-term assets can dynamically depend on the aggregate wealth of investors or intermediaries who are subject to moral-hazard constraints, but the investors in these models cannot solve moral-hazard problems by using long-term career incentives in agency contracts.

This paper may be distinguished from the previous literature by our consideration of long-term incentive contracts over more than two periods. One new result that we get from long-term contracting in this model is that the rate of growth must be gradual but contractions can be steep (see condition [5] below). This fundamental asymmetry between bounded growth rates and unbounded contraction rates could not be derived in a two-period model.

The most important goal of our model is to show how basic standard assumptions about long-term moral-hazard contracting in financial intermediation imply the general possibility of macroeconomic fluctuations. The long-term contracts that we analyze here are quite standard in the literature on dynamic moral hazard (see for example Tirole, 2006, p. 184), and the formulation here has been particularly influenced by Biais, Marrioti, Rochet, and Villeneuve (2007). (See also Myerson, 2009, for applications of such dynamic agency models to fundamental problems of government and politics.)

We assume that investors can freely recruit any number of new young bankers every period. So at any point in time there will be different cohorts of bankers of different ages who will have accumulated contractual promises and assets in long-term relationships with their investors. The aggregate values of the contractual positions of these different cohorts of mid-

As individuals are assumed to live n+1 periods, a banker can supervise an investment in each of the n periods before her last period. We are not assuming that banking requires any

The banker's expected payoff from a good project managed appropriately is αb. But if the banker instead put the investment h into a bad project, then next period she could take ηh in investment funds that were budgeted for quality verification, take an additional γh in diverted funds from the stooge-entrepreneur, and with probability β she could get a success-payment of b and demand a further kickback of e from the stooge-entrepreneur. So the banker's moral-hazard incentive constraint is

$$\alpha b \geq (\eta+\gamma)h+\beta(b+e).$$

Let us define the banker's moral-hazard coefficient

$$B = (\eta+\gamma+\beta E)/(\alpha-\beta) = \eta/(\alpha-\beta) + \gamma\alpha/(\alpha-\beta)^2.$$

These moral-hazard incentive constraints imply that minimal pay for the banker after success is

$$b = hB = h(\eta+\gamma+\beta E)/(\alpha-\beta) = h[\eta/(\alpha-\beta) + \gamma\alpha/(\alpha-\beta)^2].$$

This amount hB may be called the banker's moral-hazard rent.

We are assuming that minimal investment sizes are very large compared to the resources of a typical individual, and so the large expected moral-hazard rent αhB makes the position of banker here very attractive. But the assumption of limited resources also means that an individual cannot be asked to pay ex ante for her expected benefits of becoming a powerful banker. The moral-hazard constraint would be violated if a prospective banker raised funds for loading of rewards and have maximal punishment for failure. That is, the banker's rewards are all concentrated in one big retirement payment that depends on good performance throughout her career, but any failure will cause a termination of the banker's contract without pay, which is the worst possible punishment under limited liability. For each s in {0,1...,n–1}, the contract must specify some amount h_s that the consortium will ask the banker to invest at time t+s if her previous s investments were all successful. The contract must also specify some final payment b_n that the banker will get on retirement at time t+n if all her n investments were successful. In the Appendix we show that the optimum among such contracts is also optimal more generally in the complete class of feasible n-period contracts subject to moral-hazard incentive constraints.

At any age s in {0,...,n–1}, the banker will invest h_s at time t+s if her first s projects succeed, which with good projects has probability α^s, and so the expected time-t discounted cost of this investment at time t+s is $\alpha^s h_s/(1+\rho)^s$. The probability that the banker will make a successful investment at time t+s is α^{s+1}, and so the investors' earnings in time t+s+1, after deducting the current entrepreneur's moral-hazard rent h_sE, have the time-t expected discounted value α^{s+1} $^{s+1}$

$^n b_n/(1+\rho)^n$. Thus, at time t, the consortium's expected discounted profit (above what it could earn by lending at the global interest rate ρ) from its contractual relationship with the banker is

$$\sum_{s\in\{0,\dots,n-1\}} \alpha^{s+1}\ {}_s[r_{t+s+1} - E - (1+\rho)/\alpha]/(1+\rho)^{s+1} - \alpha^n b_n/(1+\rho)^n.$$

For good investments to be worthwhile, the rates of return r_{t+s+1} must satisfy

[3] $$r_{t+s+1} \geq E + (1+\rho)/\alpha, \quad \forall s.$$

If the rate of returns for successful investments were less than this, then the investors' expected returns, after deducting the moral-hazard rents for entrepreneurs, would be less than they could get by lending at the global risk-free interest rate ρ. Thus, investments must yield a nonnegative surplus for banking, where the banking surplus is

σ_{t+s+1} $_{t+s+1}$

$_s$

$_s$B. (As in the previous section, the banker would get 0 from failure at time t+s+1, as her contract would then be terminated without pay.) Given a successful outcome at time t+s+1, the contract offers the banker a chance of getting b_n in n–(s+1) periods if she gets n–(s+1) more successes, and this prospect has the current expected discounted value $b_n[\alpha/(1+\rho)]^{n-(s+1)}$. To satisfy the banker's moral-hazard incentive constraint at every period t+s, b_n and h_s must satisfy

$$b_n[\alpha/(1+\rho)]^{n-(s+1)} \geq h_s B.$$

Given b_n, with inequality [3], the optimal investment at each t+s is

$$h_s = b_n[\alpha/(1+\rho)]^{n-(s+1)}/B.$$

With $h_0=1$, we get

$$b_n = B[(1+\rho)/\alpha]^{n-1}, \text{ and so } h_s = [(1+\rho)/\alpha]^s, \ \forall t.$$

Thus, under the optimal contract, the amount that the banker invests will be multiplied by the factor $(1+\rho)/\alpha$ each period she succeeds. As the probability of success is α, the (unconditional) expected value of this investment is increased by the multiplicative factor $(1+\rho)$ each period during the banker's career. If the banker has success in all n periods of investment, then her consumption in retirement will equal B times the actual amount of her last investment. During a successful career, the banker's expected discounted value of this final payment grows, as the final payment becomes closer in time and more likely to be realized; and so the banker's investment responsibilities h_s grow during her career in proportion to this conditionally expected discounted value.

With this optimal plan of investments $(h_0,h_1,...h_{n-1})$ and final reward b_n, the investors' expected discounted value of profits at time t is

$$\sum_{s\in\{0,...,n-1\}} [r_{t+s+1} - E - (1+\rho)/\alpha]\alpha/(1+\rho) - B\alpha/(1+\rho).$$

We are assuming that there is a global pool of risk-neutral investors who can freely hire any number of new young bankers at any time t. So if investors could earn a strictly positive expected discounted value from such a contract, then aggregate investment in this economy would go to infinity. On the other hand, investment in this economy would vanish if such optimal contracts had a negative expected discounted value for investors. So in equilibrium with finite positive investment, the investors' optimal expected discounted value of profits must equal 0. Thus, in equilibrium, we must have

[4] $$\sum_{s\in\{0,...,n-1\}} [r_{t+s+1} - E - (1+\rho)/\alpha] = B.$$

This equation tells us that banking surpluses over the n periods of a banker's career must cover the cost of the banker's moral-hazard rents. A consortium that hired an older banker would have to distribute the same moral-hazard rents over a shorter career and so would not be profitable.

At any time $t+s+1$, for $s \in \{0,1,...,n-1\}$, if all investments so far have been successful, then latest successful investment h_s can pay the investors the dividend

$$\sigma_{t+s+1} h_s = [r_{t+s+1} - E - (1+\rho)/\alpha] h_s$$

after the entrepreneur has been paid Eh

strict inequality. Notice that, even with this modification, the expected value of the banker's investments increase by the multiplicative factor $(1+\rho)$ each period over her career.

We can now verify that, with the parametric assumption [2], the equilibrium conditions [3] and [4] imply that bad projects are unprofitable in any equilibrium. Bad projects are unprofitable at any time t when the rate of returns satisfies [1] $r_{t+1} < (1+\rho-\gamma-\eta)/\beta$. Given the definitions of B and E and $\alpha>\beta$, the inequality $(1+\rho)/\alpha+E+B < (1+\rho-\gamma-\eta)/\beta$ is equivalent to the parametric inequality [2] $\gamma/(1-\beta/\alpha)^3 + \eta/(1-\beta/\alpha)^2 < 1+\rho$.

Proposition 1

at time t will be $J_s(1+\rho)^{t-s}$. Thus, the total investment at any time t must be

$$I_t = \sum_{s\in\{t-(n-1),\ldots,t\}} J_s(1+\rho)^{t-s}, \ \forall t.$$

With these equations, $(1+\rho)I_{t-1}$ and I_t can be written as sums of terms that match for each cohort except that $(1+\rho)I_{t-1}$ includes a term $J_{t-n}(1+\rho)^n$ and I_t includes a term J_t. But in a cyclical solution, we must have $J_t = J_{t-n}$. So these equations have a cyclical solution that repeats $(J_0,\ldots,J_{n-1})$ iff

$$J_t = [(1+\rho)I_{t-1} - I_t]/[(1+\rho)^n - 1], \ \forall t.$$

The total investment of young bankers in any time period must be nonnegative. Such nonnegative J_t can be found iff the aggregate investments satisfy the inequalities

[5] $$I_t \leq (1+\rho)I_{t-1}, \ \forall t.$$

Condition [5] imposes no bound on how steeply aggregate investment can crash from one period to the next, but it tells us that aggregate investment cannot ever grow at a rate faster than ρ. Thus, our model yields a fundamental asymmetry between growth, which must be gradual, and contraction, which can be steep.

In this economy, aggregate investment I in good projects determines the rate of return for successful projects in the next period by a given investment-demand function R(I). But when $r_{t+1} = r^* = (1+\rho)/\alpha+E$, risk-free bonds with interest ρ can replace good investment projects. So we should apply an adjusted investment-demand function that does not go below r^*:

[6] $$r_{t+1} = R^*(I_t) = \max\{R(I_t), r^*\}, \forall t.$$

We can now formalize the main solution concept of this paper.

Definition. An *n-period equilibrium credit cycle* is any returns sequence $(r_1,\ldots,r_n)$ that satisfies the banking-surplus inequality [3] and the banking-rents equation [4], together with an aggregate investment sequence $(I_0,\ldots,I_{n-1})$ that cyclically satisfies the growth bounds [5] and the adjusted investment-demand equations [6].

The distinction between the investment-demand function R and the adjusted investment-demand function R^* is actually not essential to our concept of equilibrium. To see why, let I^* denote the aggregate investment such that $R(I^*)=r^*$. So I^* is the maximal investment in good projects that the economy can sustain. In any period when the bankers' contracts specify an aggregate investment I_t that exceeds I^*, the excess I_t-I^* must be invested in risk-free bonds (which investors are willing to allow, as $r_{t+1}=r^*$). But any equilibrium can be supported by an investment sequence where such bond investments do not occur. Given any equilibrium returns $(r_1,\ldots,r_n)$ where conditions [5] and [6] are cyclically satisfied by an investment sequence $(I_0,\ldots,I_{n-1})$, conditions [5] and [6] are also cyclically satisfied by $(\breve{I}_0,\ldots,\breve{I}_{n-1})$ where

$$\breve{I}_t = \min\{I_t, I^*\}, \ \forall t.$$

In effect, the transformation from I_t to $\breve{I}_t$ shifts the recruitment of some young bankers earlier in time across periods when the banking surplus rates σ_{t+1} are 0.

Assuming that bankers are hired with efficient long-term contracts, as described in the previous section, the dynamic state of the economy at any point in time will depend on its history d8.04 0 ugh

[8] $\sum_{t\in\{0,\ldots,n-1\}} [R(\sum_{s\in\{0,\ldots,n-1\}} J_{\tau+t-s}(1+\rho)^{s}) - E - (1+\rho)/\alpha] < B$ and $J_{\tau} = 0$

for all τ such that $0 \le \tau \le T-1$,

$\sum_{t\in\{0,\ldots,n-1\}} [R(\sum_{s\in\{0,\ldots,n-1\}} J_{T+t-s}(1+\rho)^{s}) - E - (1+\rho)/\alpha] = B$,

$J_T \ge 0$, and $J_t = J_{t-n}$ for all t σσσσσρσ career at time t with no capital, $k_0 = 0$. To invest h_s at time t+s, a banker with capital k_s must borrow $h_s – k_s$, and the banker must promise to repay her risk-neutral ρ-discounting investors the amount $(h_s – k_s)(1+\rho)/\alpha$ at time t+s+1, in the α-probability event of success. Thus, success at time t+s+1 will yield banker's capital

$$k_{s+1} = [r_{t+s+1} – E – (1+\rho)/\alpha]h_s + k_s(1+\rho)/\alpha = \sigma_{t+s+1}h_s + k_s(1+\rho)/\alpha.$$

With $k_0 = 0$ and $h_s = h_0[(1+\rho)/\alpha]^s$, induction yields the equations

$k_s = h_{s–}$ T0)(1+1 1.4 TD()Tj F 1 Tf 0.6.04 0 0 8.04 36.39966769 1 tf0 T(s)Tj F5 1)]TJ 4 12 0 0 12 24.99965409 TT7(s)0.0001 1 12 0+... 0 TD()Tj F 87Tf 0.

moral hazard in banking, financial regulation may be needed for two reasons: to assure investors that a banker's capital must be legitimately earned, and to assure investors that a banker's investments do not exceed the appropriate age-dependent multiple of her legitimate capital.

Under this alternative system of financial governance, the dynamic state of the economy would be defined in terms of the accumulated capital of each cohort of bankers. The total investment that can be made by a cohort with a given stock of accumulated capital would depend on the anticipated returns in future periods. A version of Proposition 2 for a financial system with independent regulated bankers would be somewhat more complicated and is omitted here.

In the rest of this paper, our analysis assumes a financial system where bankers are hired by consortiums of investors with long-term contracts, as in Section IV.

V. x p r r upp r ur ur .( )

equilibria can range between the minimum of $r^* = (1+\rho)/\alpha+E = 1.301$ and the maximum possible rate of $(1+\rho)/\alpha+E+B = 1.689$.

It may be helpful to see how the steady-state equilibrium depends on the bankers' career length n. In a steady-state n-period equilibrium cycle, the banking surplus is always $\sigma = B/n$, and so, with the parameters in [9], and the constant rate of return on successful investments is

$$r = (1+\rho)/\alpha + E + B/n = 1.301 + 0.388/n.$$

With this rate of return, aggregate investment is

$$I = (\psi-r)/(\pi\alpha) = 1.983 - 1.751/n,$$

and expected total net product (less the cost of invested inputs with interest), is

$$Y = [\alpha\psi-(1+\rho)]I = 1.097 - 0.968/n.$$

In this net product, the total wage income for workers is

$$W = \pi(\alpha I)^2 = 0.828 - 1.461/n + 0.645/n^2,$$

the total income for entrepreneurs is

$$E\alpha I_t = 0.269 - 0.238/n,$$

and the total profit for bankers is

$$[r_{t+1}-E-(1+\rho)/\alpha]\alpha I_t = (B/n)\alpha I_t = 0.731/n - 0.645/n^2.$$

cohorts' investments plus the new-bankers' investment J.

V. x p r

Now let us analyze some dynamic equilibria, assuming that bankers are hired with efficient long-term contracts, as described in Section IV. Consider the example with parameters as above in [9] with n=10, but suppose that the continuing bankers' investments at time 0 are 80% of the steady state amounts in line [10] above. Such a situation could occur if the economy was previously in steady state, but then an unanticipated technical change at time 0 increased investment demand by a permanent 20% reduction of the parameter π (to 0.233). So at time 0, the total contractually-mandated investments $\theta_s(0)$ for continuing bankers of each age s are

$$(\theta_1(0)...,\theta_9(0)) = (0.100, 0.110, 0.121, 0.133, 0.146, 0.161, 0.177, 0.195, 0.214).$$

Each of these current-investment amounts corresponds to an initial investment of $J_{-s} = \theta_s(0)/(1+\rho)^s = 0.091$ for s = 1,...,n–1.

To compute the equilibrium that evolves from these initial conditions, we only need to find J_0, the total investments that new bankers make at time 0. The contractual investments of each cohort grow by the multiplicative factor $(1+\rho)$ each period until the cohort retires at age n, and then it must be replaced by new cohort whose new investments will equal the final investment of the old retiring cohort divided by $(1+\rho)^{n-1}$, so that the new cohort will repeat the retiring cohort's investments n periods later. Any increase of J_0 would increase all future investments I_t and so would decrease all future returns $r_{t+1}=R^*(I_t)$. In equilibrium, we must have $\sigma_1+...+\sigma_{10} = B$, as in equation [7], this equation here has the solution $J_0 = 0.318$. The resulting 10-period equilibrium credit cycle is shown in Figures 1 and 2.

[Insert Figures 1 and 2 about here]

In this equilibrium, the shortage of bankers at time 0 causes a large cohort of new age-0 bankers to enter and handle investment J_0

dj /F1 1 Tf 12 0 0 12 83.321976466.1539 T [(= 0....399e s)

than in the steady state, reaching a peak at time 10, with output 9.6% above steady state and returns $r_{10} = 1.301$.

At time 10, the generation-0 bankers retire and consume their accumulated profits, thus creating a new scarcity of investment intermediaries. Then investment at time 10 drops in a recession to the same level as at time 0, and the cycle repeats itself.

The workers't aazbs W

Other examples can be found where stabilization subsidies are not worth the expense for tax-paying workers, however, and it seems difficult to characterize the cases where it is worthwhile. But we can make broader statements about another kind of financial stimulus that is simpler to analyze.

Suppose that we are given an n-period equilibrium credit cycle where, at time 0, continuing bankers of any age $s \in \{1,...,n-$

period (that is, it is the steady-state investment for n=1 with all other parameters unchanged).

$J_{n-s} = J_{-s}$ for s = 1,...,n–1. In a cyclical equilibrium, the resulting investments must yield return rates with banking surpluses that just cover the bankers' moral-hazard rent B over the next n periods, as in condition [7] above. But condition [7] cannot be satisfied for this example, because any nonnegative J_0 will yield investment returns in times 1 through n that are too low:

$$\sum_{t\in\{0,\ldots,n-1\}} [R(\sum_{s\in\{0,\ldots,n-1\}} J_{t-s}(1+\rho)^s) - E - (1+\rho)/\alpha] < B, \quad \forall J_0 \geq 0.$$

That is, the continuing contractual investments at time 0 are too high to admit any profitable investment by new bankers at time 0. Thus we must have $J_0 = 0$ in a dynamic equilibrium here.

The failure to satisfy the banking-rents equation [4] at time 0 means that initial conditions cannot be part of a cyclical equilibrium, and so the cohort that retires at time 1 can be replaced by a new cohort that is smaller. That is, J_1 here may different from J_{-9}, and instead we can choose J_1 to start a new cyclical equilibrium with $(J_{-8},J_{-7},\ldots,J_0)$, by satisfying the equation

$$\sum_{t\in\{1,\ldots,n\}} [R(\sum_{s\in\{0,\ldots,n-1\}} J_{t-s}(1+\rho)^s) - E - (1+\rho)/\alpha] = B$$

with $J_t = J_{t-n}$ for all $t\geq 2$. This is solved by $J_1 = 0.045$, satisfying condition [8] with T=1.

[Insert Figure 3 about here]

Thus, the vector $(\theta_1(0)\ldots,\theta_9(0))$ here yields a dynamic equilibrium which begins with one transient period at time 0, when returns are too high for any new bankers to enter, and thereafter it evolves as an n-period equilibrium credit cycle repeating the sequence of investments and returns from times 1 to 10. Figure 3 shows the distribution of investments across age cohorts and time in this equilibrium. In the bar at time 0, the shaded parts indicate the pattern of investments that is repeated at time 10 and every 10th period thereafter, and the white rectangle at the top of the bar indicates an additional investment (0.214) by old bankers at time 0 that is not repeated by old bankers at time 10 or thereafter. Aggregate investment declines slowly from time 0 to time 9. Thereafter, in each subsequent pass through the 10-period cycle, the economy grows strongly in the first two periods, as the small cohorts retire, but then the economy drops into another long slow recessionhe 10f0.1(y)ic te of Japan's lost decade after the collapse of the 1980s boom.

Finally, let us consider what would happen in the worst-case scenario when the economy starts at time 0 with no bankers at all, so that $\theta_t(0)=0$ for all t. From this initial condition, with the parameters above in [9], the equilibrium credit cycle has initial investment $J_0 = I_0 = 1.338$. Investment then grows at the maximal rate ρ for 4 periods and thereafter levels off at the peak $I_t = 1.983$ from time t=5 onwards, with no new entry of bankers until the generation-0 bankers retire at period n. Thus, output at the trough in this worst-case scenario is 33% less than output at the peak, which takes 5 periods to reach from the trough. Remarkably, this result does not depend on the parameter n, as long as n>5. That is, the potential depth and duration of recessions in our model do not depend on the length of the bankers' careers.

[illegible]

Financial crises and recessions are vast complex phenomena, but their inexorable momentum must be derived from factors that are fundamental in economic systems. Theoretical financial recovery must drive gradually uphill into the next boom, when the economy will have an excess of bankers relative to what can be sustained in the steady state, and this boom can in turn contain the seeds of a future recession.

A stabilization that shifts the economy from such a recession to the steady state would require some new investments to be handled by older bankers who are more expensive, because their moral-hazard rents cannot be distributed over as many periods of future investment. Investors would be unwilling to use these costly shorter-term intermediaries without a subsidy. But we found that, in some cases, the workers' benefits from such macroeconomic stabilization may be greater than the cost of the required subsidies. In this sense, a tax on poor workers to subsidize rich bankers may actually benefit the workers, as the increase of investment and employment can raise their wages by more than the cost of the tax. Some of these wage increases, however, would come at the expense of other investors who must re-invest past earnings under previously negotiated financial contracts.

This paper is part of a growing theoretical literature on the important question of how macroeconomic instability may be derived from incentive constraints in microeconomic transactions, and more models are needed. The model here has made many simplifying assumptions which should be relaxed in future research.

pp x R ur r u r p r r

Consider a contractual relationship, at time t, between a consortium of investors and a banker who started at time 0. Let y_t denote the value at time t of rewards that were previously promised to the banker by the consortium. Let m_{t+1} denote the expected marginal cost to the consortium at time t+1 of increasing the banker's expected future rewards by one unit of value at time t+1. Rewards cannot be deferred at time n, so $m_n = 1$. At time 0, the investors have made no prior promise to the young banker, and so $y_0 = 0$.

In the contract, let h_t denote the size of their investment at time t. For the cases of success and failure, respectively, let e and f denote payments to the entrepreneur, and let b and c denote the value of rewards to the banker at time t+1. Then the consortium's optimization problem at time t is:

choose $h_t \geq 0$, $b \geq 0$, $c \geq 0$, $e \geq 0$, and $f \geq 0$ so as to

maximize $\alpha(r_{t+1}h_t - e - m_{t+1}b) - (1-\alpha)(f + m_{t+1}c) - (1+\rho)h_t$

subject to [*Lagrange multipliers*]

$\alpha e + (1-\alpha)f \geq \gamma h_t + \beta e + (1-\beta)f,$ $[\lambda_e]$

$\alpha b + (1-\alpha)c \geq (\gamma+\eta)h_t + \beta(b + e) + (1-\beta)(c + f),$ $[\lambda_b]$

$\alpha b + (1-\alpha)c \geq (1+\rho)y_t.$ $[\mu_t]$

If $\sigma_{t+1} > Bm_{t+1}$ then infinite solutions would be feasible with $c=0$, $f=0$, $e=Eh_t$, $b=Bh_t$, taking $h_t \to +\infty$. So we must have $m_{t+1} \geq \sigma_{t+1}/B$. Then we can show that the optimal solution is

$$c = 0,\ f = 0,\ e = h_t E,\ b = h_t B,\ \text{and}\ h_t = y_t(1+\rho)/(\alpha B).$$

This solution satisfies the three constraints with equality, and it maximizes the Lagrangean with multipliers:

$$\lambda_e = (\alpha+\lambda_b\beta)/(\alpha-\beta),\ \lambda_b = \alpha\sigma_{t+1}/[(\alpha-\beta)B],\ \mu_t = m_{t+1} - \sigma_{t+1}/B.$$

These make e, h_t, and b drop out of the Lagrangean, which becomes

$$-\mu_t(1+\rho)y_t - c\sigma_{t+1}/B - (1+\lambda_b)f.$$

Thus, in the optimal solution (with c=f=0) the consortium at time t+1 has an expected net cost

$$\mu_t(1+\rho)y_t = (m_{t+1} - \sigma_{t+1}/B)(1+\rho)y_t.$$

This expected cost is linear in the previously promised rewards y_t, and this linearity can recursively justify our assumption of linear costs for future promised rewards b and c. A unit increase in y_t would increase the consortium's expected cost at time t+1 by $(1+\rho)\mu_t$, and so it would increase the consortium's expected cost at time t by μ_t. So the Lagrange multiplier μ_t that we get from the above problem is equal to the parameter m_t that is the marginal cost of rewards promised to the banker at time t, for the consortium's analogous investment problem at time t–1:

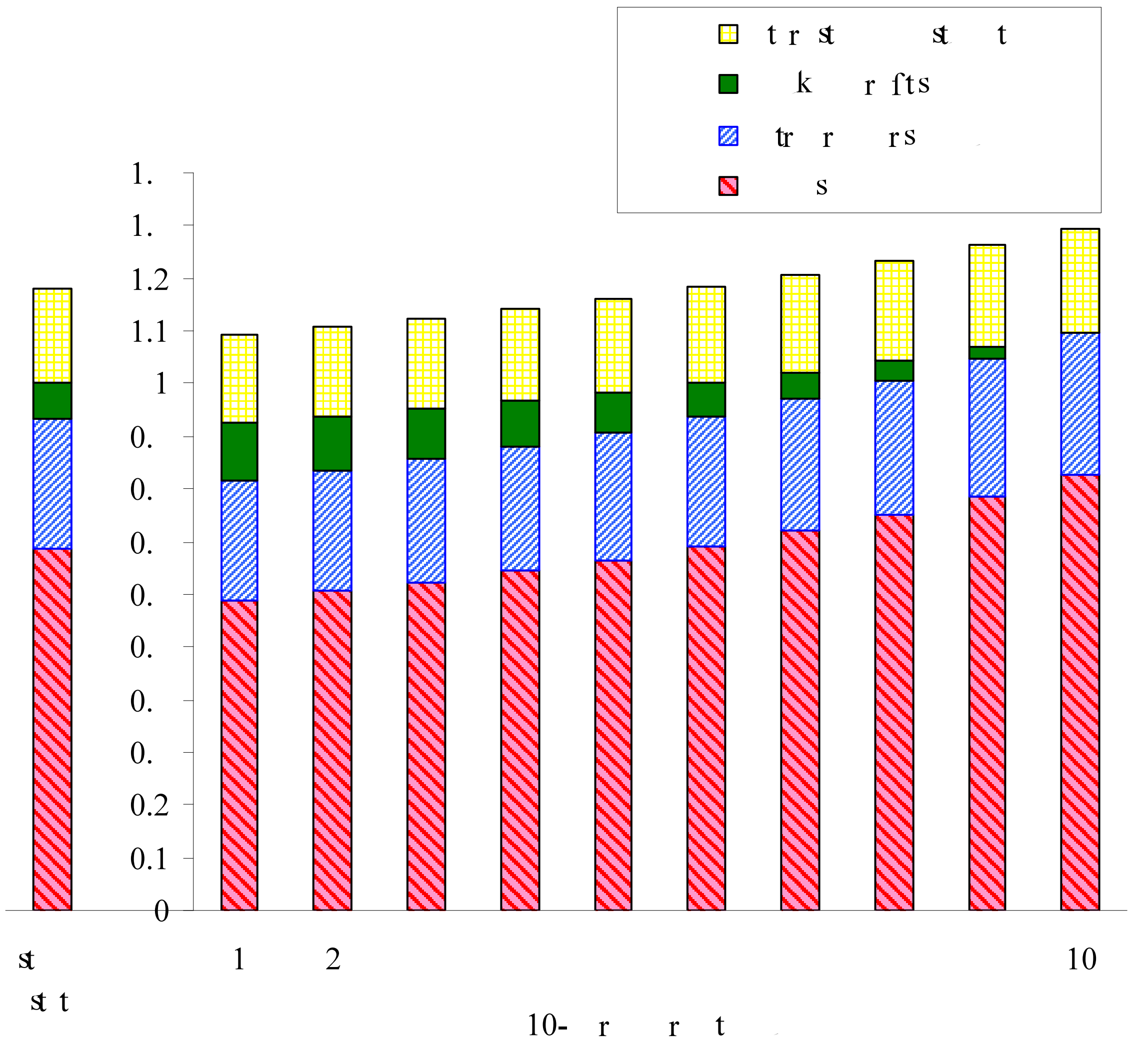
$$m_t = \mu_t = m_{t+1} - \sigma_{t+1}/B.$$

Thus, with $m_n = 1$, we get by induction:

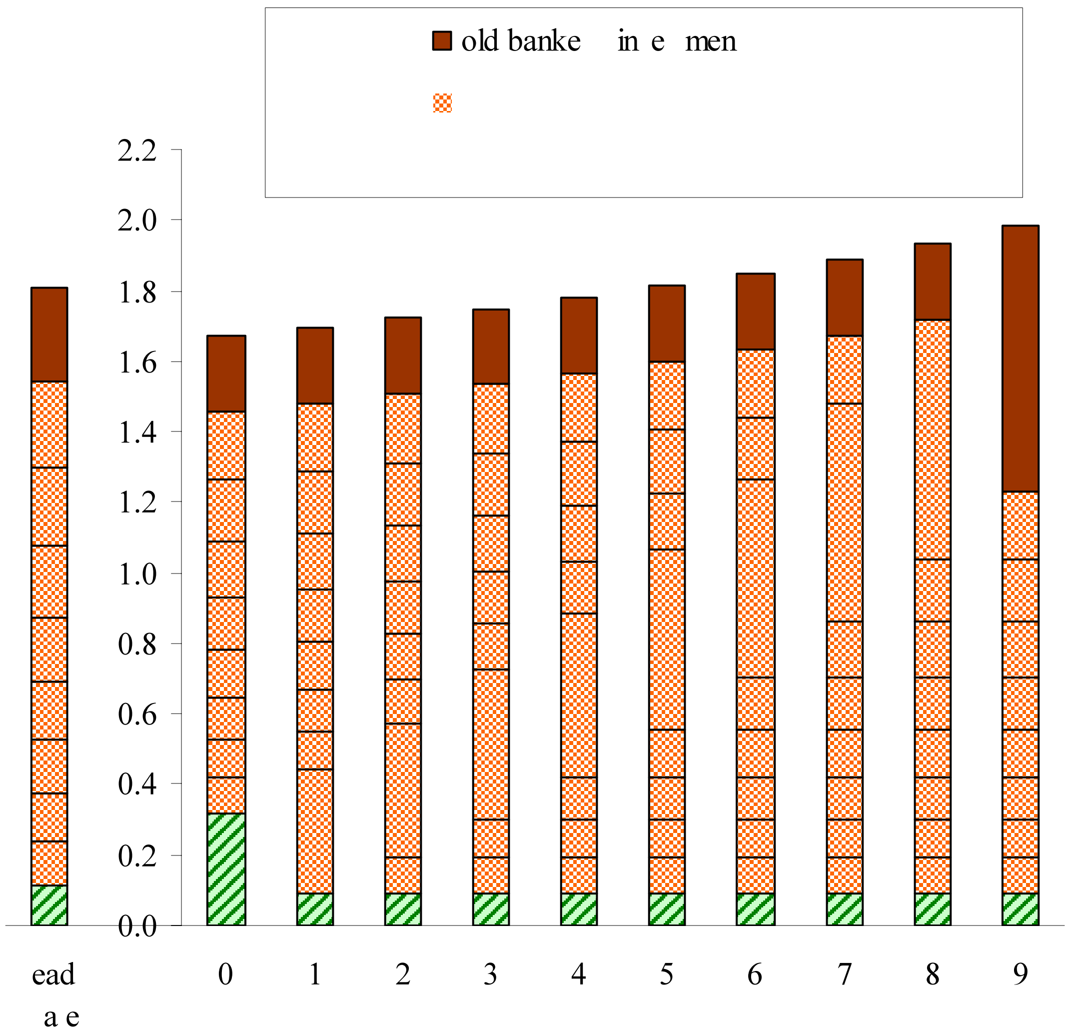
$$m_t = \mu_t = 1 - (\sigma_{t+1}+...+\sigma_n)/B,\ \ \forall t \in \{1,...,n-1\}.$$

If successful at time t+1, the banker will be promised $y_{t+1} = b = h_t B$, and her next investment will be $h_{t+1} = h_t B(1+\rho)/(\alpha B) = h_t(1+\rho)/\alpha$. Thus, success multiplies her investment by $(1+\rho)/\alpha$ each period.

At time t=0, we have $y_0=0$, but then a solution $e = h_0 E$, $b = h_0 B$, and $h_0 > 0$ can be optimal for the consortium, with slack in the promise-keeping constraint, if and only if this constraint has multiplier $\mu_0 = 0$, which is equivalent to the banking-rents equation [4].



ur 1. Net product in a 10-period credit cycle, with continuing bankers' investments at time 0 being 80% of steady state. Parameters: ρ=0.1, n=10, α=0.95, β=0.6, γ=0.05, η=0, ψ=1.74, π = 0.233.


old banke in e men
2.2
2.0
1.8
1.6
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0.0
ead
a e
0
1
2
3
4
5
6
7
8
9

R . r .

Costas Azariadis and Bruce Smith, "Financial intermediation and regime switching in business cycles," *American Economic Review* 88(3):516-536 (1998).

Gary Becker and George Stigler, "Law enforcement, malfeasance, and compensation of enforcers," *Journal of Legal Studies* 3:1-18 (1974).

Ben Bernanke and Mark Gertler, "Agency costs, net worth, and business fluctuations," *American Economic Review* 79(1):14-31 (1989).

Bruno Biais, Thomas Mariotti, Jean-Charles Rochet, Stephane Villeneuve, "Large risks, limited liability, and dynamic moral hazard," *Econometrica* 78(1):73-118 (2010).

Markus K. Brunnermeier and Yuliy Sannikov, "A macroeconomic model with a financial sector," Princeton University working paper (2009).

Douglas W. Diamond, "Financial intermediation and delegated monitoring," *Review of Economic Studies* 51:393-414 (1984).

Nicholas Figueroa and Oksana Leukhina, "Business cycles and lending standards," University of Washington working paper (2010).

Xavier Freixas and Jean-Charles Rochet, *Microeconomics of BankingteaT-1 an-321 of of]TJ3 hicholas-1o eor-. Eicsmic Studies*

Carl Shapiro and Joseph E. Stiglitz, "Equilibrium unemployment as a worker disciplinary device," *American Economic Review* 74:433-444 (1984).

Javier Suarez and Oren Sussman, "Endogenous cycles in a Stiglitz-Weiss economy," *Journal of Economic Theory* 76:47-71 (1997).

Javier Suarez and Oren Sussman, "Financial distress, bankruptcy law and the business cycle," *Annals of Finance* 3:5-35 (2007).

Jean Tirole, *The Theory of Corporate Finance*, Princeton University Press (2006).

AUTHOR'S ADDRESS:
Roger Myerson, Economics Department, University of Chicago, 1126 East 59th Street, Chicago, IL 60637.
Phone: 1-773-834-9071. Fax: 1-773-702-8490. Email: myerson@uchicago.edu
Web site: http://home.uchicago.edu/~rmyerson/
This paper: http://home.uchicago.edu/~rmyerson/research/bankers.pdf
Notes: http://home.uchicago.edu/~rmyerson/research/banknts.pdf
[current version: Feb 7, 2011]