Nonparametric Estimation of Expected Shortfall

ON X C[illegible]

Io[illegible] St[illegible]te [illegible]niversity

ABSTRACT [illegible]

1. INTRODUCTION

e e pec ed o f nd e e e pop e e of n nc fo n e o po fo o of e A zne De en e nd e o c e dd y p ope y n ene nd ence no co e en e e n con co e en o e nd c ed nd eco e o e c e e n e n n nc n e en

Le $\{X_t\}_{t=1}^{n}$ e e e e of n e o po fo o of e o e n pe od of e n Le Y_t $-log X_{it}/X_{it-1}$ e e ne e o e n o o o e e t pe od ppo e $\{Y_t\}_{j=1}^{n}$ dependen on y p oce e n d on f nc on F en po e e p c o e o ze o e con dence e e $-p$

$$\nu_p \quad \text{nf}\{y \quad F(y) \geq \quad -p\}$$

c e $-p$ q n e of e o e d on F e pec e e e of nonp e c e ne e o nd pp e fo en y n y n e con e of po fo o oc on

An d n e of e nonp e c e od e n ode f ee nd ence ode o nd o d c ed y n pec ed o d on n nc n e en p y conce ned c ce c of p of e o d on o e d e ene y p e n e nd ence p opo n p ope p e c o ode c deq e fo e p no e e nonp e c e od c n p y n c n o e Ano e d n e of e nonp e c pp o c o de n e of d dependence c e d p e n e con e of n nc o e e nonp e c e o con de ed n p pe c n cco o d e d dependence e p c y nce e e ec of e dependence on e nce of e e on c n e c e y pe o n e nce fo d e en f o e e e e e pp o c e e e ec e y e e ceed nce ndependen nd den c y d on o e on c e y p o c y nde e o c ed D nd D' cond on Le d e e L nd en nd oo zen An e p c dy y Be n nd nc y c y n o nonp e c n e o n n nc e n c n e on dependence e en fo e e o d e e nd c e e need fo con de n e dependence n n nc e n d ec y c e pp o c en y e nonp e c e o con de ed n p pe

n p pe e e e o nonp e c e o One ed on e ed p e e e of e ce e o e de ned y e o ν_p ed on n o de c Ano e e e ne e o p opo ed n c e c e p oy e ne oo n n o e n e on nd e n e n of e e ce e o e e ope e e ne oo n o d e e e on o e cc e o e ed fo e c e of e on y C en nd n o e o n y e e e nce nd e e n q e e o of e e ne e o no nece y e n of p e e e e o A e fo e on of e e p e p e e e of e ce e o e ood eno nd e e no need o c y o e ne oo n y e p n con de n e ne oo n e d o e nce n q n e e on fo o ndependen e e nd M on nd dependen C en nd n o e on nde y n e on fo o nn e e d e en e ec of e ne oo n y on e f c e e ec e y e n p e e c c n e e ed e cc e y y p e e n

e p pe c ed fo o e n od ce e o nonp e c e o n ec on e c p ope e e d c ed n ec on ec on epo on e c fo o ed y n e p c dy on o n nc e e n ec on A e ec n c de e en n e ppend

2. NONPARAMETRIC ESTIMATORS

A p e nonp e c e o of e e ed e e of e ce e o e e n ν_p e e ν_p $Y_{([n(1-p)]+1)}$ e p e q n e e o of ν_p nd $Y_{(r)}$ e r o de c of $\{Y_t\}_{t=1}^{n}$ en e

$$\mu_p \quad \frac{\sum_{t=1}^{n} Y_t I Y_t \geq \nu_p}{\sum_{t=1}^{n} I Y_t \geq \nu_p} \quad np \quad {}^{-1} \sum_{t=1}^{n} Y_t I Y_t \geq \nu_p$$

e e I e nd c o f nc on

e e ne e o p opo ed y c e e fo o n Le K e e ne f nc on c y e c p o y den y f nc on nd $G t$ $\int_t^{\infty} K y\, dy$ nd $G_h t$ $G t/$ e e po e oo n nd d e e ne e o of e f nc on $S x$ $- F x$

$$S_h z \quad n^{-1} \sum_{t=1}^{n} G_h z - Y_t$$

A e ne e o of ν_p deno ed $\nu_{p,h}$ e o on of $S_h z$ p p opo ed n o e o L en nd c e [2] By ep c n e nd c o f nc on nd ν_p e oo e G_h nd $\nu_{p,h}$ e pec e y n c e p opo ed e fo o n e ne e o

$$\mu_{p,h} \quad np \;{}^{-1} \sum_{t=1}^{n} Y_t G_h \nu_{p,h} - Y_t \;.$$

[2]Its statistical properties and how to obtain the standard errors are considered in Chen and Tang (2005). See also Cai (2002) and Fan and Gu (2003) for ke(n)(e)(l)(t)(c)(1)(T)il5 0 Td 5 2p

B ed on e p o e en of e e ne e o $\nu_{p,h}$ o e ν_p e pec ed e e ne e o $\mu_{p,h}$ o d p o e e e on cc cy of e n oo ed e o μ_p Con n o o e e e foc of e ne ec on

3. MAIN RESULTS

e p ope e of e e o nonp e c e o e e ed n ec on e o e cond on

Le $\mathcal{F}_k^l$ e e σ e of e en ene ed y $\{Y_t, k \leq t \leq l\}$ fo $l > k$ e α n coe c en n od ced y o en

$$\alpha k \quad \underset{A \in \mathcal{F}_1^i, B \in \mathcal{F}_{i+k}^\infty}{\text{p}} |P AB - P A P B|.$$

e e e d o e α n f $_{k\to\infty}$ αk e dependence de c ed y e α n e e e p ed y o e ype of n ee Do n fo co p e en e d c on e fo o n cond on e ed n o dy

e e e $\rho \in$, c $\alpha k \leq C\rho^k$ fo $k \geq$ nd po e con n C

e d on F of Y_t o e y con n o p o y den y f c con n o econd de e n $\mathcal{B}\nu_p$ ne o ood of ν_p e o n d on f nc on of Y_1, Y_{k+1} F_k fo $k \geq$ e econd p de e o nded n $\mathcal{B}\nu_p$ $E|Y_t|^{2+\delta} \leq C$ fo o e $\delta >$ nd po e con n C

K y e c p o y den y fy n e o en cond on $\int_{-1}^{1} u K u \, du$ nd $\int_{-1}^{1} u^2 K u \, du$ $\sigma_K^2 >$ nd K o nded nd L p c z con n o de e e $\rightarrow$ n $^{3-\beta} \rightarrow \infty$ fo ny $\beta >$ nd n ^{4} o $^2 n$ $\rightarrow$ $n \rightarrow \infty$

Cond on e n e e e e eo e c α n c ed y ny co on y ed n nc e e e c nc de e A MA A C e oc c o y nd d on ode o n nce M y nd e e ed e α n fo A C de nd enon C o e n e nd L edo fo d on ode Cond on con n nd d cond on c eq e nde y n oo ne fo e n nd p e o n den e o e e n e o en fo e o e e n Cond on nd e e one eq ed y e e ne e o e

Cond on eq e en on e e ne Cond on pec e n e fo e nd d c nc de $O(n^{-1/3})$ e op o de fo e n e on e e cond on e co p e cond on po ed y o e o o n nce Cond on nd co e pond o A p on nd f A p on nd c nd of c e e pec e y

Le $\gamma(k)$ $Cov\{(Y_1 - \nu_p) I(Y_1 \geq \nu_p), (Y_{k+1} - \nu_p) I(Y_{k+1} \geq \nu_p)\}$ fo po e n e e k nd

$$\sigma_0^2(p, n) \quad \{Var\{(Y_1 - \nu_p) I(Y_1 \geq \nu_p)\} \quad \sum_{k=1}^{n-1} \gamma(k)\}.$$

A p on nd e D ydo neq y p y $\sigma_0^2(p, n)$ n e fo e c n nd con e n $n \to \infty$

e e n e n oo ed e o μ_p o p o de po n of efe ence fo e e ne e o De on en n e ppend o nde cond on nd nd fo n y po e κ

$$\hat{\mu}_p - \mu_p \quad p^{-1}\{n^{-1}\sum_{i=1}^{n}(Y_t - \nu_p) I(Y_t \geq \nu_p) - p(\mu_p - \nu_p)\} \quad o_p(n^{-3/4+\kappa}).$$

B d ype e p n on c e d o e fo o n eo e e d n e y p o c no y of $\hat{\mu}_p$

M nde cond on $n \to \infty$

$$\sqrt{pn}\,\sigma_0^{-1}(p, n)(\hat{\mu}_p - \mu_p) \xrightarrow{d} N(\ ,\ ).$$

e eo e nd c e e y p o c nce of $\hat{\mu}_p$ $\sigma_0^2(p, n)/(np)$ c e nce of $p^{-1}\{n^{-1}\sum_{i=1}^{n}(Y_t - \nu_p) I(Y_t \geq \nu_p) - p(\mu_p - \nu_p)\}$ e e d n o de e n e e p n on e dependence n e o n e e e e ec ed n e y p o c nce o e co nce e n $\sigma_0^2(p, n)$ e n e need o cco od e e dependence n f e c nfe ence n e e on nd ypo e e n fo e o f e on e no e o e e ec e p e ze fo e e on on y np A p n n e een o 0 nd o 0 co on y ed n nc n e en e e o ec o o y c co on c en e fo c nfe ence of e e

e fo o n eo e ze e p ope e of e e ne e o
M nde cond on $n \to \infty$

$$\sqrt{pn}\sigma_0^{-1} p \; n \; \mu_{p,h} - \mu_p \xrightarrow{d} N ,$$

nd f e o e

$$Bias \; \mu_{p,h} \quad -\tfrac{1}{2}p^{-1}\sigma_K^2 \; {}^2 f \; \nu_p \quad o \; {}^2 \quad \text{nd}$$

$$Var \; \mu_{p,h} \quad p^{-1}n^{-1}\sigma_0^2 p \; n \quad o \; n^{-1} \; .$$

By co p n eo e fo nd e e ne e o e e y p o c no d on e n oo ed p e e o μ_p o e co e pond n e fo e on epo ed n C en nd n e o no e o μ_p nd $\mu_{p,h}$ con e e o μ_p e e of $\sqrt{n}$ o o e p ec e y e e of $\sqrt{np}$ e e e e o ν_p nd $\nu_{p,h}$ con e e o ν_p e e of $\sqrt{n}$ o o e p ec e y e e of $\sqrt{n} f \nu_p$ e e f e p o y den y of Y_t

e econd p of e eo e con ey d e en o y f o e on of n e e e on e e ne e o doe no o e nce ed c on e econd o de of n^{-1} e econd o de e n e A e e e e oo n n n n c ed o n o e nc e e n e e n q e e o e efo e fo e p po e of e n e e e ne oo n co n e p od c e nde y n e on e f c e e ec e y e n p e e c c n e e ed e cc e y o oo n e on o nonp e c e on of e e n p e e c c n e e e ed y e p e e n

o d e no ed e o e e en on y pp c e fo po n e on of o con c n con dence n e nd e n ypo e on μ_p n e p e ence of d dependence e e ne oo n c n p y n c n o e n e n $\sigma_0^2 p \; n$ e pec den y e on pp o c

4. SIMULATION STUDY

n ec on e epo e fo on dy c e e e pe fo nce of e nonp e c e o e n o ec e o con o eo e c nd n n e p e o ec on e ode c o en fo e o o Y_t n e on e

n

e ne e on of e pec den y of

p $n/(2q)$ nd $\nu^2(q)$ $_{0\le j\le 2q-1} E\left(\sum_{l=[jp]+1}^{[(j+1)p]} X_l\right)^2$ o n eq y en n Yo oy ( ) $\nu^2(q) \le Cp$ App y eo e n Bo q ( ) fo α n eq ence

$$P\{|F_n(\nu_p \quad \epsilon_n) - F(\nu_p \quad \epsilon_n)| > C_1\epsilon_n\}$$
$$\le \quad \text{e p}\left(-\frac{C_1^2\epsilon_n^2 q}{\sigma^2(q)}\right) \quad \{ \quad \frac{}{C_1\epsilon_n}\}^{1/2} q\alpha\{ n/(2q) \} \qquad (A \;)$$

e e $\sigma^2(q)$ $p^{-2}\nu^2(q)$ ϵ_n $C\epsilon_n$ o o

$$\text{e p}\left(-\frac{C_1^2\epsilon_n^2 q}{\sigma^2(q)}\right) \le \quad \text{e p}\{-C_2\epsilon_n q\} \qquad (A \;)$$

e e $C_2 >$ nce $n\epsilon_n^2 \to \infty$ e n $q\epsilon_n \to \infty$ e e n (A con e e o ze o e ponen y f On e econd e of (A e eo e c α n p e

$$\{ \quad \left(\frac{}{C_1\epsilon_n}\right)^{1/2} q\alpha\{ n/(2q) \} \le C\epsilon_n^{-1/2} q\rho^{[n^{1/2}\log^{-1}(n)/2]} \qquad (A \;)$$

c con e e o ze o e ponen y f oo co p e e e p oof of Le □

L MMA nde e cond on ( ) ( ) nd fo ny $\kappa >$

$$n^{-1}\sum(Y_t - \nu_p)\{I(Y_t \ge \hat\nu_p) - I(Y_t \ge \nu_p)\} \quad o_p(n^{-3/4+\kappa}).$$

OO Le W_t $(Y_t - \nu_p)\{I(Y_t \ge \hat\nu_p) - I(Y_t \ge \nu_p)\}$ e e e $E(W_t)$ No e $E(W_t)$ $-I_{t1}$ I_{t2} e e

$$I_{t1} \quad E\{(Y_t - \nu_p) I(\nu_p \le Y_t < \hat\nu_p) I(\hat\nu_p > \nu_p)\} \quad \text{nd}$$
$$I_{t2} \quad E\{(Y_t - \nu_p) I(\hat\nu_p \le Y_t < \nu_p) I(\hat\nu_p < \nu_p)\}.$$

e o e e I_{t1} I_{t11} I_{t12} nd I_{t2} I_{t21} I_{t22} e e fo $a \in (\;, \; / \;)$ nd $\eta >$

$$I_{t11} \quad E\{(Y_t - \nu_p) I(\nu_p \le Y_t < \hat\nu_p) I(\hat\nu_p \ge \nu_p \quad n^{-a}\eta)\},$$
$$I_{t12} \quad E\{(Y_t - \nu_p) I(\nu_p \le Y_t < \hat\nu_p) I(\nu_p < \hat\nu_p < \nu_p \quad n^{-a}\eta)\},$$
$$I_{t21} \quad E\{(Y_t - \nu_p) I(\nu_p > Y_t \ge \hat\nu_p) I(\hat\nu_p \le \nu_p - n^{-a}\eta)\} \quad \text{nd}$$
$$I_{t22} \quad E\{(Y_t - \nu_p) I(\nu_p > Y_t \ge \hat\nu_p) I(\nu_p > \hat\nu_p > \nu_p - n^{-a}\eta)\}.$$

App y n e C c y z neq y fo k nd

$$|I_{tk1}| \le \sqrt{E(\hat\nu_p - \nu_p)^2 P(|\hat\nu_p - \nu_p| \ge n^{-a}\eta)}.$$

en Le nd e f c $E(\nu_p - \nu_p)^2$ $O(n^{-1})$ p y

$$I_{tk1} \to$$ e ponen y f . (A

o e e I_{t12} e no e $|I_{t12}| \leq E\{(Y_t - \nu_p) I(\nu_p \leq Y_t < \nu_p \quad n^{-a}\eta)\}$. e n

$$I_{t12} \leq \int_{\nu_p}^{\nu_p + n^{-a}\eta} d(z - \nu_p) f(z)\, dz \quad O(n^{-2a}) .$$

n e e c y e pp o c e c n o I_{t22} $O(n^{-2a})$ e e e nd (A e n y c oo n a $-$ / γ e e $\gamma >$ y

$$E(W_t) \quad o(n^{-1+\kappa})$$ (A

fo n y po e κ c n n p e

$$E\Big[n^{-1} \sum (Y_t - \nu_p)\{I(Y_t \geq \nu_p) - I(Y_t \geq \nu_p)\}\Big] \quad o(n^{-1+\kappa}) .$$ (A

e no con de $Var(W_i)$ o $a \in ($, /

$$\begin{aligned}
E(W_t^2) \quad & E\Big[(Y_t - \nu_p)^2\{I(Y_t \geq \nu_p) - I(Y_t \geq \nu_p) I(Y_t \geq \nu_p) \quad I(Y_t \geq \nu_p)\}\Big] \\
& E\Big[(Y_t - \nu_p)^2\{I(\nu_p > Y_t \geq \nu_p) \quad I(\nu_p > Y_t \geq \nu_p)\}\Big] \\
& E\Big[(Y_t - \nu_p)^2 I(\nu_p \leq Y_t < \nu_p)\{I(\nu_p \geq \nu_p - n^{-a}\eta) \quad I(\nu_p < \nu_{pn}{}^{-a}\eta \ a_\eta)\}\Big] \\
& E\Big[(Y_t - \nu_p)^2 I(\nu_p > Y_t \geq \nu_p)\{I(\nu_p \geq \nu_p \quad n^{-a}\eta) \quad I(\nu_p < \nu_p \quad n^{-a}\eta)\}\Big].
\end{aligned}$$

No e

$$E\{I(\nu_p \leq Y_t < \nu_p) I(\nu_p \leq \nu_p - n^{-a}\eta)\} \leq P(|\nu_p - \nu_p| \geq n^{-a}\eta) \quad \text{nd}$$
$$E\{I(\nu_p > Y_t \geq \nu_p) I(\nu_p > \nu_p \quad n^{-a}\eta)\} \leq P(|\nu_p - \nu_p| \geq n^{-a}\eta)$$

c con e e o ze o e ponen y f p ed y Le App y n e C c y c z neq y e e

$$E\{(Y_t - \nu_p)^2 I(\nu_p \leq Y_t < \nu_p) I(\nu_p \leq \nu_p - n^{-a}\eta)\} \quad \text{nd}$$
$$E\{(Y_t - \nu_p)^2 I(\nu_p > Y_t \geq \nu_p) I(\nu_p \geq \nu_p \quad n^{-a}\eta)\}$$

con e e o ze o e ponen y f e en pp y n e e e od e
A e e

$$E\{(Y_t - \nu_p)^2 I(\nu_p \le Y_t < \nu_p) I(\nu_p \ge \nu_p - n^{-a}\eta)\} \quad O(n^{-3a}) \quad \text{nd}$$

$$E\{(Y_t - \nu_p)^2 I(\nu_p > Y_t \ge \nu_p) I(\nu_p < \nu_p \quad n^{-a}\eta)\} \quad O(n^{-3a}).$$

n y e e $E(W_t^2)$ $o(n^{-3/2+\kappa})$ nd A e n $Var(W_t)$ $o(n^{-3/2+\kappa})$
By y od fy n e o e de on fo $Var(W_t)$ y e o n fo ny t_1, t_2
$Cov(W_{t_1}, W_{t_2})$ $o(n^{-3/2+\kappa})$ e efo e

$$Var\left[n^{-1}\sum_{i=1}^{n}(Y_t - \nu_p)\{I(Y_t \ge \mu_p) - I(Y_t \ge \nu_p)\}\right] \quad o(n^{-3/2+\kappa}). \qquad (A$$

o e e A e d y e e e

L MMA Le β $(np)^{-1}\sum Y_t G_h(\nu_p - Y_t)$ nd η $(n)^{-1}\sum_{i=1}^{n} Y_t K_h(\nu_p - Y_t)$ nde e
cond on

$$(\ )\quad Cov\left[\beta, \{p - S_h(\nu_p)\}\{f(\nu_p) - f(\nu_p)\}\right] \quad o(n^{-1}),$$

$$(\ )\quad Cov\left[\beta, (\eta - \eta)\{p - S_h(\nu_p)\}\right] \quad o(n^{-1}),$$

$$(c)\quad Cov\left[\{p - S_h(\nu_p)\}, (\eta - \eta)\{p - S_h(\nu_p)\}\right] \quad o(n^{-1}).$$

OO e on y p e en e p oof of e p oof fo e o e e De ne
β $E(\beta)$ Le $\beta - \beta$ $n^{-1}\sum \psi_1(Y_t)$ $f(\nu_p) - f(\nu_p)$ $n^{-1}\sum \psi_2(Y_t)$ $O(\ )^2$ nd $p - F_h(\nu_p)$
$n^{-1}\sum \psi_3(Y_t)$ $O(\ )^2$ fo o e f nc on ψ_j j , nd c $E\{\psi_j(Y_t)\}$ o
n nce $\psi_2(Y_t)$ $K_h(\nu_p - Y_t) - E\{K_h(\nu_p - Y_t)\}$ nd $\psi_3(Y_t)$ $G_h(\nu_p - Y_t) - E\{G_h(\nu_p - Y_t)\}$
n e pp o c n B n ey p

$$A \quad |E\left[(\beta - \beta)\{p - S_h(\nu_p)\}\{f(\nu_p) - f(\nu_p)\}\right]$$

$$\le \quad n^{-2} \sum_{i \ge 1, j \ge 1, i+j \le n} |E\{\psi_1(Y_1)\psi_2(Y_i)\psi_3(Y_{i+j})\}| \qquad O(n^{-1}\ ^4 \quad n^{-2}\ ^2) \qquad (A$$

e e nd c e e d e en pe on on e ee nd ce Le p δ
q δ nd s^{-1} $- p^{-1} - q^{-1}$ fo o e po e δ o e D ydo neq y

$$|E\{\psi_1(Y_1)\psi_2(Y_i)\psi_3(Y_{i+j})\}| \le \quad ||\psi(Z_1)||_p ||\psi_2(Y_t)\psi_3(Z_{i+j})||_q \alpha^{1/s}(\ ).$$

e e o e p oy n e o

By c oo n r n^a fo $a \in$, nd k' n^c c $c \in$, $-a$ e c n o e ef nd de of A con e e o $n \to \infty$ en e

$$ {}_n \xrightarrow{p} \qquad n \to \infty. \qquad \text{A}$$

e efo e $S_{n,1}$ $n^{-1/2} \sum_{j=1}^{r} W_{j,n}$ o_p

By pp y n e neq y e ed n Yo oy nd e con c on of $W_{j,n}$ e

Le η $E\{p^{-1}Y_tK_h Y_t - \nu_p\}$ $p^{-1}\int \nu_p - u\, K u\, f \nu_p - u\, du$ $p^{-1}\nu_p f \nu_p$ O^{2} n nd d de on fo α n eq ence fo n nce o e en n Bo q e e $Cov\{ np^{-1}\sum Y_tK_h \nu_p - Y_t, \nu_{p,h} - \nu_p\}$ $O n^{-1}$ [redacted] e f o A

$$E\{ np^{-1}\sum Y_tK \nu_p - Y_t \nu_{p,h} - \nu_p\} \qquad \eta E \nu_{p,h} - \nu_p \quad O n^{-1}$$

$$-\tfrac{1}{2}p^{-1}\nu_p f' \nu_p \;^{2}\sigma_K^2 \quad \alpha^{3} \quad O n^{-1} \quad A$$

Co ne A A nd A

$$E \mu_{p,h} \qquad \mu_p - \tfrac{1}{2}p^{-1}\sigma_K^2 \;^{2} f \nu_p \quad \alpha^{2} \quad O n^{-1}$$

c e e e en n

e no de e e nce of $\mu_{p,h}$ Le A_1 $np^{-1}\sum_{t=1}^{n}\{Y_tG_h \nu_p - Y_t - Y_tK_h \nu_p - Y_t \nu_{p,h} - \nu_p\}$ e e e d n o de e of e e p n on A

en

$$Var A_1 \qquad Var\{ np^{-1}\sum Y_tG_h \nu_p - Y_t\} \quad Var\{\eta \nu_{p,h} - \nu_p\}$$

$$- \quad Cov\{ np^{-1}\sum Y_tG_h \nu_p - Y_t, \eta \nu_{p,h} - \nu_p\}. \qquad A$$

e y o ee

$$Var\{ np^{-1}\sum Y_tG_h \nu_p - Y_t\}$$

$$n^{-1}p^{-2}\Big[Var\{Y_tG_h \nu_p - Y_t\} \quad \sum_{k=1}^{n-1} - k/n\; Cov\{Y_1G_h \nu_p - Y_1, Y_{k+1}G_h \nu_p - Y_{k+1}\Big].$$

Le c_K $\int_{-\infty}^{\infty} uK u\, du \int_{-\infty}^{u} K v\, dv$ y e o n

$$Var\{Y_tG_h \nu_p - Y_t\} \qquad \int z^2G_h^2 \nu_p - z\, f z\, dz - p^2\mu_p^2 \quad O^{2}$$

$$\int_{-\infty}^{\infty} K u\, du\Big[\int_{-\infty}^{u} K v\, dv\{\int_{\nu_p}^{\infty} z^2 f z\, dz \quad \int_{\nu_p - hv}^{\nu_p} z^2 f z\, dz\}$$

$$\int_{u}^{\infty} K v\, dv\{\int_{\nu_p}^{\infty} z^2 f z\, dz \quad \int_{\nu_p - hu}^{\nu_p} z^2 f z\, dz\} - p^2\mu_p^2 \quad O^{2}$$

$$Var\{Y_tI Y_t \geq \nu_p\} - \nu_p^2 f \nu_p\, c_K \quad O^{2}. \qquad A$$

q on A nd Le e n

$$Var\{ np^{-1}\sum Y_tG_h \nu_p - Y_t\} \quad p^{-2}Var\{\phi_1 \nu_p\} - n^{-1}\;\nu_p^2 f \nu_p\, c_K \quad \alpha n^{-1}. \quad A$$

e econd e on e nd de of A

$$Var\{\eta \nu_{p,h} - \nu_p \} \quad \eta - \eta \nu_{p,h} - \nu_p \}$$
$$\eta^2 Var \nu_{p,h} \quad \eta Co \nu_{p,h}, \eta - \eta \nu_{p,h} - \nu_p \} \quad Var\{ \eta - \eta \nu_{p,h} - \nu_p \}.$$

y e o n y n e f c η $p^{-1}\nu_p f \nu_p$ O ^{2}

$$\eta^2 Var \nu_{p,h} \quad p^{-2}\nu_p^2 Var\{n^{-1}\sum_{t=1}^{n} I Y_t > \nu_p \} - p^{-2}n^{-1}b\nu_p^2 f \nu_p \, c_K \quad n^{-1}b \,. \qquad A$$

o e neq y en n Yo oy fo α n eq ence

$$E \nu_{p,h} - \nu_p \;^4 \leq Cn^{-2} \quad \text{nd} \quad E \eta - \eta \;^4 \quad O n^{-2 \;\; -3} \,.$$

App y n e C c y c z neq y nd Le

$$Var\{ \eta - \eta \nu_{p,h} - \nu_p \} \quad O n^{-2 \;\; -3/2} \quad n^{-1} \qquad \text{nd} \qquad A$$
$$Co\{\eta \nu_{p,h} - \nu_p \,, p^{-1} \eta - \eta \nu_{p,h} - \nu_p \} \quad n^{-1} \quad . \qquad A$$

Co ne A A nd A

$$Var\{\eta \nu_{p,h} - \nu_p \} \quad p^{-2}\nu_p^2 Var\{\phi_2 \nu_p \} - p^{-2}n^{-1} \;\nu_p^2 f \nu_p \, c_K \quad n^{-1} \quad . \qquad A$$

o Le e co nce e on e nd de of A

$$Co\{ np \;^{-1}\sum_{t=1}^{n} Y_t G_h \nu_p - Y_t \,, \eta \nu_{p,h} - \nu_p \}$$
$$Co\{ np \;^{-1}\sum_{t=1}^{n} Y_t G_h \nu_p - Y_t \,, \eta f^{-1} \nu_p \; n^{-1}\sum_{t=1}^{n} G_h \nu_p - Y_t \} \quad n^{-1}$$
$$np^2 \;^{-1}\nu_p \Big[Co\{Y_t G_h \nu_p - Y_t \,, G_h \nu_p - Y_t \}$$
$$\sum_{k=1}^{n-1} - k/n \; Co\{Y_1 G_h \nu_p - Y_1 \,, G_h \nu_p - Y_{k+1} \}\Big] \quad n^{-1}$$

nce $Co\{Y_t G_h \nu_p - Y_t \,, G_h \nu_p - Y_t \}$ $p - p \;\; p - \nu_p f \nu_p \;\; c_K$

$$Co\{ np \;^{-1}\sum_{t=1}^{n} Y_t G_h \nu_p - Y_t \,, np \;^{-1}\sum_{i=1}^{n} Y_t K_h \nu_p - Y_t \nu_{p,h} - \nu_p \} \qquad A$$
$$n^{-1}p^{-2}\nu_p Co\{\phi_1 \nu_p \,, \phi_2 \nu_p \} - n^{-1}p^{-2}\nu_p^2 f \nu_p \quad c_K \quad .$$

e A A nd A o A e no e e econd o de e of $O(n^{-1})$ c nce o e c o e nd e efo e

$$Var(\mu_p) \quad p^{-1}n^{-1}\sigma_0^2(p,n) \quad o(n^{-1}) \; , \qquad (A$$

c e

e y p o c n B y of $\nu_{p,h}$ c n e e ed f o A y n e e oc n e od n e p oof of eo e

NC

A zne De en e M nd e D Co e en e e of *M the tic Fin nce* **9**

Be n nd nc De ec n nd ode n dependence M n c p nd p pe p e en ed n n e Me od n nce Confe ence C n A ch ece e c d e d e ene Be n ey nd McNe A nd e pec ed o f n po fo o of dependen c ed concep nd p c c n *ourn of B nking nd Fin nce*

enon C o e n e nd C L edo oc c o y Mode d en M o Mode nd c App c on *Bernoulli*

o e o C c e O nd L en en y n y of e *ourn of e piric n nce*

o on *e t Risk* nd d on Ne Yo Mc

Le d e e M L nd en nd oo zen *E tre es nd Re ted Properties of R ndo Sequences nd Processes* p n e Be n

M y nd e D Nonp e c e on nd den c on of non ne c e e e *Econo etric Theory* **11**

e D nd o M *St tistic An ysis of E tre e es ith pp i c tions to insur nce n nce hydro gy nd other e ds nd ed* Bo on B e e

o en M A cen eo e nd e α n cond on *Proc N t Ac d Sc S A* **42**

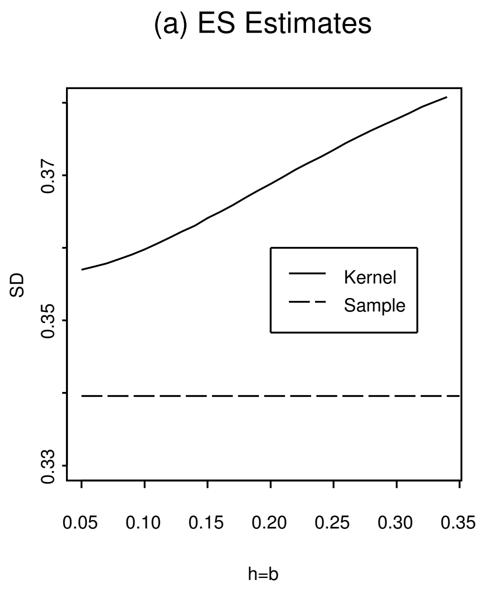
c e O Nonp e c e on of cond on e pec ed o f en e e eo

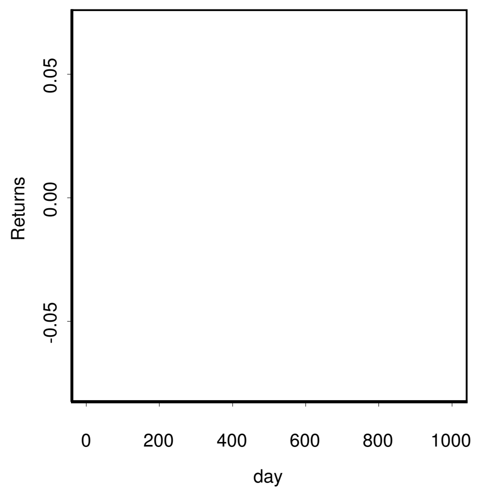
c e O Nonp e c e on nd en y n y of e pec ed o f *M the tic Fin nce* **14**

e e nd M on e ne q n e e o *ourn of A eric n St tistic Associ tion* **5**

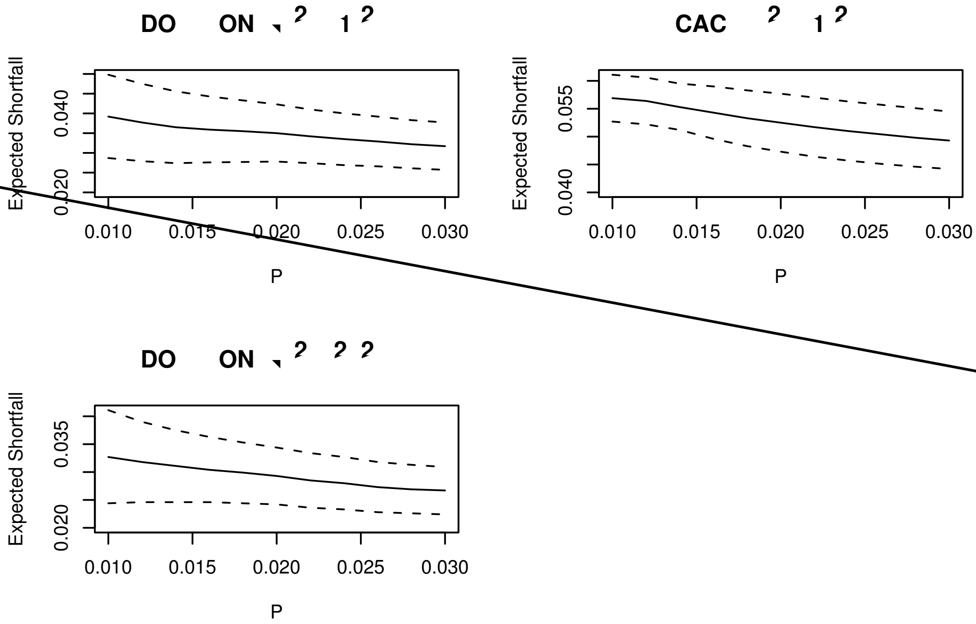
Yo oy Mo en o nd fo on y n eq ence *Pro i ity Theory nd Re ted Fie ds* **52**

Yo e B d ep e en on of p e q n e fo eq ence of on y n ndo e *St tistics nd Pro i ity Letters* **24**


(a) ES Estimates
SD
0.33
0.35
0.37
0.05
0.10
0.15
0.20
0.25
0.30
0.35
h=b
Kernel
Sample



Returns
0.05
0.00
-0.05
0
200
400
600
800
1000
day


DO ON ? 1 ?
Expected Shortfall
0.020
0.040
0.010 0.015 0.020 0.025 0.030
P
CAC ? 1 ?
Expected Shortfall
0.040
0.055
0.010 0.015 0.020 0.025 0.030
P
DO ON ? ? ?
Expected Shortfall
0.020
0.035
0.010 0.015 0.020 0.025 0.030
P

T Esti tes for $\nu_{0.01}$ $\mu_{0.01}$ nd St nd rd Errors S.E

	CAC		Do one	
Ye	$\nu_{0.01}$	μ_p	$\nu_{0.01}$	μ_p