Endogenous Exporting Decisions of Heterogeneous Firms: Theory and Evidences

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Abstract

The paper attempts to explore exporting decisions of heterogeneous firms by endogenously determining both the number of exporting destinations/countries, and the number of varieties for differentiated goods, which is, as we know so far, the first paper to explicitly provide both numbers simultaneously in the literature. Extending from the Melitz's framework (Melitz, 2003), it endogenizes the number of exporting destinations and the number of varieties for differentiated goods for a heterogeneous firm, while providing the cutoff value of transportation cost for the firm's exporting that is different from Melitz's (2003) and other related literatures. Moreover, it shows such numbers are constant at equilibria and they converge to these upper bounds with productivity we find.

In addition, while we show such functions of variety and exporting destinations with respect to productivity are concave, the critique values to ensure the concavity are constant as well and same for both functions, which is surprisingly interesting. Our empirical tests prove theoretical results above as well.

Keywords: Heterogeneous firms, productivity, spectrum of exporting destinations, number of differentiated goods

JEL Classification: F12, F23

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1. Introduction

The performance of a heterogeneous firm, especially multinational corporations (i.e. MNCs), on its decision about exporting and/or other decision concerning its exporting has attracted increasing interests widely. For example, people could wonder how many differentiated goods it plans to produce, which should relate to its decision of the number of products. On the other hand, while Melitz (2003) and others initiate the research on heterogeneous firms and

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other hand, however, there is relatively little theoretical research examines how firms determine the spectrum of countries they will export to. That is we expect to explore in this paper.

In relevant literature, one finds that when firms export, they typically export multiple product, they find a positive and significant correlation between the number of products that firms export and the number of countries they export to (Bernard, Jensen, Redding, and Schott, 2007). However, it is only empirical results, without any explicit solution for them. Moreover, they donot explore the relationship and any analytical solution for the number of differentiated goods within one product and the number of exporting destinations, or exporting countries assuming one destination is one country for simplicity in this paper.

Another strand of the literature documents and interprets the relationship between firms' productivity levels and the collection of foreign markets that they serve (Eaton, Kortum, and Kramarz, 2004 and 2007). These papers find that most exporting firms sell to only one foreign market, with the frequency of firms' selling to multiple markets declining with the number of destinations. At the same time, firms selling to only a small number of markets tend to sell to the most popular ones. Less popular markets are served by firms that export very widely. These patterns are consistent with the notion that firms with relatively low marginal costs can profitably exploit relatively more foreign markets.

Here, we only discuss the condition that all heterogeneous firms in the same industry. So, we can think all the firms produce one kind of product, but there are different brands, as differentiated goods, within the type of product.

The breath of exporting markets is assumed as the number of exporting countries, as we assumed.

2. Models

2.1. Demand

The preferences of a representative consumer are given by Melitz (2003) as a C.E.S. utility function:

$$U = \left[\int_{\omega\in\Omega} q(\omega)^{\rho}\, d\omega\right]^{\frac{1}{\rho}}, \quad 0 < \rho < 1,$$

Where φ and φ_D are productivity of the firm's brands and the cutoff productivity of this industry for domestic market. These brands are substitutes, that implies and $0 < \rho < 1$ an elasticity of substitution between any two brands of $\sigma = \frac{1}{1-\rho}$.

Suppose that the producer is also a consumer, this means that a representative consumer has disposable income R. Where $R = PQ = \int_{\omega\in\Omega} p(\omega) q(\omega)\, d\omega$ represents aggregate expenditure of the firm, P is an aggregate price of the industry.

The consumer attempts to

$$\max_{q(\omega)} U \quad \text{s. t.} \quad \int_{\omega\in\Omega} p(\omega) q(\omega) d\omega = R, \quad \omega \in \Omega$$

This provides these following results

$$q(\omega) = Q\left[\frac{p(\omega)}{P}\right]^{-\sigma},$$

Where Q is equal to U.

$$P = \left[\int_{\omega\in\Omega} p(\omega)^{1-\sigma} d\omega\right]^{\frac{1}{1-\sigma}},$$

$$r(\omega) = R\left[\frac{p(\omega)}{P}\right]^{1-\sigma},$$

Where $R = PQ = \int_{\omega\in\Omega} r(\omega) d\omega$ also denotes total revenue of this firm.

2.2. Production

Production also attempts to adopt Melitz (2003) as follows: there is only one factor, labor, which is inelasticity supplied. But, here, we divide the labor into two sorts: skilled labor and unskilled workers. Moreover, we assume that organizing the brands requires skilled labor. Production is assumed to be easy and could be accomplished by unskilled workers. Organizing is deemed to be characterized by the diminishing marginal return. Specifically, the organization cost for a certain number n of brands is $w \cdot n^m$, $m > 1$, where w means each organizer's wage. The m represents the marginal technology in this country, which is exogenous to firms. To enter the industry each firm has to paying a fixed cost F, moreover, the first brand's entry cost is f_e and the sum of n brands is $n^m f_e$ for entering the market. The marginal firm is defined as $n(\omega_E) = 1$

Unskilled labor function is

$$l \quad \frac{q}{N},$$

This yields a pricing rule

$$p \quad M \quad \frac{1}{UM},$$

Then firm profit is

think the firm treats those countries who it choose to export equally, that is this firm exports the same quality and number of brands to those countries, however, due to different market size of these countries, the exporting quantity maybe different. The marginal condition means $S(\varphi_E) = 0$, where φ_E is the cutoff productivity for export about these firms, which belong to this industry.

Then, when this firm exports its products, it chooses to produce a certain number of brands n and spectrum of exporting countries S for maximization of its profit. That is:

$$\max_{S,n} \sum_{i=1}^{n} \int_{\varphi_D}^{\varphi_M} \mu(\varphi)\left[1 - S(\varphi)^{1-\sigma}\right]_i v \, d\varphi - \left[w n^m + n^m f_e + S n^m f_e + F(\varphi)\right] = \varphi$$

Moreover,

$$\sum_{i=1}^{n} \int_{\varphi_D}^{\varphi_M} \mu(\varphi)\left[S(v(\varphi))\right]_i d\varphi = \int_{\varphi_D}^{\varphi_M} \mu(\varphi)\left[\sum_{i=1}^{n} v_i(\varphi)\right] d\varphi = \int \mu(\varphi) S(\varphi)\, \varphi \, d\varphi$$

$$= \frac{MR}{B\varphi(\sigma)} \int_{\varphi_D}^{\varphi_M} \varphi^{1-\sigma} d\varphi$$

So the number of brands is a function of productivity.

Then, the above question can be transformed as the following problem:

$$\max_{S,\varphi} \sum_{i=1}^{n} \int_{\varphi_D}^{\varphi_M} \mu(\varphi)\left[1 - S(\varphi)^{1-\sigma}\right]_i v \, d\varphi - \left[w n^m + n^m f_e + S n^m f_e + F(\varphi)\right] = \varphi$$

Through first order conditions of this problem, we can get an analytical expression of S (see Appendix A):

$$S = \frac{MR \tilde{A}(\varphi)}{B\varphi(\sigma) W f_e} - \frac{W w - f_e}{f_e}\left[1 - \frac{1}{\ldots}\frac{w - f_e}{W f_e}\right],$$

Where $A(\varphi)$ means the definite integral $\int_{\varphi_D}^{\varphi_M} \varphi^{1-\sigma} d\varphi$, $\tilde{A}(\varphi)$ is the productivity in equilibrium, which is higher than the cutoff productivity for exporting; B is equal to the improper integral $\overset{f}{\underset{M_D}{}} M^{1} e^{V} \, d M \quad {}^{J} \quad {}^{M}$

Where $\bar{S}$ is a constant in equilibrium .

$$\lim_{M \to \infty} S = \bar{S}.$$

That is, S converges to $\bar{S}$ with development of productivity and $\bar{S}$ is the boundary of S.

Proposition 1 *The spectrum of exporting countries for a firm is the function of its productivity, and there is an upper bound for the spectrum which is constant. Furthermore, wwitith development of its productivity, the*

, t is boundary finallty

spectrum of exporting countries to rise, so both partial derivatives of spectrum with respect to average wage and total revenue are all positive. But the increase of transaction cost and the first brand's entry cost induce contrary change in its profit, reducing its exporting spectrum.

The curve of $S\quad S\quad M$:

$$S\,c = \frac{MR\,\tilde{}\,M^{1}e^{V}}{BM\;V\,f_e W} M^{V} \frac{w^{J}\;f_e{}^{M}}{f_e} \;!\;0, \qquad W$$

This means the spectrum of exporting countries increases with productivity.

$$S\,cc = \frac{1}{J}\,M \S \frac{MR\,V\;{}^{1}e^{V}}{\copyright\;BM\;V\,f_e W}\;{}^{V}\frac{w^{J}\;f_e{}^{M}\tilde{}}{f_e} M, \qquad M \qquad W$$

$$S\,cc\,M \circledR \begin{cases} 0, & {}_{E}!\;M \quad \frac{V\;1}{J} M \text{ convex;} \qquad d \\ 0, & M \quad \frac{V\;1}{J}, \text{ concave.} \qquad ! \end{cases}$$

Where M_E is the exporting cutoff productivity for this industry and we assume that M_E is less than $\frac{V\;1}{J}$.

Figure1 The shape of curve $S \quad S \quad M$

Then we have

Proposition 2 *There is a value* $\frac{V\ 1}{J}$*, if the productivity level is between* M_E *and* $\frac{V\ 1}{J}$*, the curve of* $S \quad S \quad M$ *is convex; if higher than it, the curve is concave.*

Average wage of the firm:

$$\bar{w} \quad {}_1 .1 \quad O_2 w, \qquad O_1 \quad 0,1 \ , \ {}_2 \quad 0,1 \ , \qquad \bullet O$$

Then

$$\frac{S}{\bar{w}} \quad \frac{wS}{w} \frac{w}{w\bar{w}} \quad \frac{1w}{\varsigma_2} \frac{S}{ww} \quad 0 \ \frac{w}{w} \qquad \varsigma_2 \bullet \frac{w}{w} 0,1 \ !$$

Where ς_1 *and* ς_2 *are the proportions of unskilled and skilled labor to the firm's total workers respectively.*

Remarks: From the above, we can see that the spectrum of exporting countries also increases with average wage of this firm.

4. Variety of differentiated goods

Firm entry and exit

The marginal condition is defined as that a firm only produces one kind of product ($n \quad M_D \quad 1$) and whose net profit after organization cost is just enough to cover the fixed cost F .

Then the zero profit conditions are given as the following under different situations:

Closed economy $\quad {}_1 S_D \quad M F \quad w,$

Export $\quad S_1 \quad {}_E \quad F \quad w \; M f_e, \qquad W$

Figure 2 Zero profit condition

Where M_E is the cutoff productivity of export firms for the industry.

Free entry condition

$$\frac{1 \;\; G \;\; M}{G} S_1 \quad f_e \quad 0,$$

$$S_1 \quad \frac{G f_e}{1 \;\; G \;\; M},$$

Where $1 \;\; G \;\; M_D$ is the ex ante probability of drawing a productivity above the zero-value cutoff M_D upon entry into the industry. When the firm does produce, G is a constant probability which the firm faces in every period of a bad shock that would force it to exit.

4.1 Closed Economy

As we discussed before, the representative firm expects to maximize its profit with respect to variety of this firm:

$$\max_{n} \sum_{i=1}^{n} 3_{M_3}^{M} Mu \; ª\neg M_i \;\; v \;\; S \;\; d^{º}¼ \; M n^{m} \; ª n^{m}\neg M_e \;\; F, \;\; M \;\; º \; ¼$$

Where $u\ \ M v\ \ M$, being equal to $\mathcal{L}^{JM}$, is the probability of drawing productivity M upon entry into the market.

Moreover,

$$\sum_{i=1}^{n}{}_{D} \; Mu \;\; M ª\neg_i \; S \;\; v \;\; M \;\; d^{º}¼ \; 3_{M_D}^{M} \; Mu \; ª\neg M v \; \sum_{i=1}^{n}{}_{i} \; M^{º} d \; ¼ \;\; M \;\; S \;\; \neg M \;\; M$$

$$\frac{MR}{BM\ V} 3_{M_3}^{M} \; {}^{1}e^{V} \; dM. \;\; ^{J} \;\; ^{M} \;\; M$$

So the variety is a function of productivity. The problem of maximization of profit with respect to variety can be transformed the following issue:

$$\max_{M} \sum_{i=1}^{n} 3_{M_3}^{M} Mu \; ª\neg M_i \;\; v \;\; S \;\; d^{º}¼ \; M n^{m} \; ª n^{m}\neg M_e \;\; F, \;\; M \;\; º \; ¼$$

Then we obtain the following result (see Appendix D for proof)

$$n \;\; \frac{MR\ \tilde{A}\ \ M^{*}}{BM\ V\ \ w^{\sim}\ f_e} \;\; 1« \; ª^{\frac{1}{m}} \; « \;\; º \; » \; ¼$$

Where $A\ \ M$ means the definite integral $3_{M_3}^{M} M^{1} e^{V} \, d\, M, \;\; ^{J*} \;\; ^{M}$

For a certain firm, larger marginal technology m would lead to higher organizational cost, which reduces the firm' profit gain and then the quantity of its number of varieties.

$$\frac{N}{w} \quad \frac{MR}{m\,V\ w \quad f_e\ ^2}\ \frac{MR\ \tilde{A}}{BM\ V\ w \quad f_e}\ 1^{a\,\frac{1}{m}\ 1}\ \frac{A\,M}{BM} \quad 0$$

$$\frac{2}{2} \quad 1 \quad \frac{\frac{1 \quad {}^{1} \quad {}^{1} \quad {}^{1} \quad {}^{2}}{1}}{{}^{2} \quad \frac{1 \quad {}^{1}}{1} \quad 1^{2\ \frac{1}{}}}$$

$$\frac{1}{} \quad \frac{1 \quad {}^{1} \quad {}^{1}}{1 \quad \frac{1 \quad {}^{1}}{1} \quad 1^{1\ \frac{1}{}}}.$$

$$\frac{\partial n}{\partial m} = \frac{\ln\left(\frac{AMR(1-S)\tau^{\sigma}}{BM\,(w-(1-S)\tau)\,f_e}\right)}{\left(\frac{AMR(1-S)\tau^{\sigma}}{BM\,(w-(1-S)\tau)\,f_e}\right)^{\frac{1}{m}} m^2} \quad 0$$

$$\frac{\partial n}{\partial \tau} = \frac{\frac{AMR\left[(1-S)\sigma(w-S\tau)\tau^{\sigma-1}f_e + (1-S)^{1-\sigma}\tau^{\sigma} f_e\right]}{BM\,m\,(w-(1-S)\tau)^2 f_e^2}}{\left(\frac{AMR(1-S)\tau^{\sigma}}{BM\,(w-(1-S)\tau)\,f_e}\right)^{1-\frac{1}{m}}} \quad 0,$$

When exporting to other countries, an increase in transaction cost τ can bring higher total cost to a firm, this reduces the firm' profit, so the number of brands will become fewer.

$$\frac{\partial n}{\partial w} = \frac{\frac{AMR(1-S)\tau^{\sigma}}{BM\,m\,(w-(1-S)\tau)^2 f_e}}{\left(\frac{AMR(1-S)\tau^{\sigma}}{BM\,(w-(1-S)\tau)\,f_e}\right)^{1-\frac{1}{m}}} \quad 0$$

Average wage for the firm:

$$\bar{w} = \alpha_1 \cdot 1 + \alpha_2 w, \quad \alpha_1 \in (0,1), \quad \alpha_2 \in (0,1)$$

Then

$$\frac{\partial n}{\partial \bar{w}} = \frac{\partial n}{\partial w}\frac{\partial w}{\partial \bar{w}} = \frac{1}{\alpha_2}\frac{\partial n}{\partial w} \quad 0, \quad \alpha_2 \in (0,1).$$

Where α_1 and α_2 are the proportions of unskilled and skilled labor to the firm's total workers respectively.

$$\frac{\partial n}{\partial S} = \frac{\frac{AMR^{1}\,(w\,f_e - \tau f_e)^{\sigma}}{BM\,m\,(w-(S-1)\tau)^2 f_e^2}}{\left(\frac{AMR(1-S)^{1}}{BM\,(w-(1-S)\tau)\,f_e}\right)^{\frac{1}{m}-1}}$$

$$\begin{cases} \dfrac{\partial W}{\partial \delta} > 0, & \text{if } \tau < \left(\dfrac{w - f_e}{f_e}\right)^{\frac{1}{\sigma}}, \\ \dfrac{\partial W}{\partial \delta} < 0, & \tau > \left(\dfrac{w - f_e}{f_e}\right)^{\frac{1}{\sigma}}. \end{cases}$$

Proposition 4: *When a firm exports, there is a value* $\left(\frac{w - f_e}{f_e}\right)^{\frac{1}{\sigma}}$, *if the transaction cost is lower than it, the number of varieties increase with exporting countries; if higher than it, the number would decrease.*

For proposition 4, the existing value $\left(\frac{w - f_e}{f_e}\right)^{\frac{1}{\sigma}}$ is determined by skilled labor's wage, first variety's entry cost and elasticity of substitution between any two goods. But, it is a constant in the equilibrium, because the average wage would be a constant at that time.

If the transaction cost is lower than it, all the exporting varieties are profitable, then the firm has more richer capital to introduce new varieties, so the number of total product varieties would increase with exporting countries; if higher than it, a few of exporting varieties would be losing gradually. Moreover, with the increase of exporting countries the competition would become fiercer, particularly the average wage has a trend of increase with time, all these factors result in the reduction of number.

5. Empirical tests

5.1 Exporting destinations

Assume that the first variety's entry cost and the transaction cost are all constant. We can get the empirical model

$$S_{it} \qquad {}_1R_{it} \quad E_2 \quad {}_{it} \qquad {}_3\overline{w}_{it} \; E u_i + v_{it} \, , \; M \qquad\qquad E$$

Where λ is measured in terms of either revenue-based labor productivity or TFPR, here we choose revenue-based measures of productivity, but, not quantity-based measures of productivity[3], because data on physical units of output is not available for all products, moreover, physical units are not comparable across firms for many products, e.g., cars, motorcycles etc; company is denoted by i and t means year. u_i captures the company fixed effects and v_{it} is a stochastic error.

The dataset in this research is constructed for 74 motorcycle firms of China for 7-year period from 2000 to 2006. This 7 years period is between Asian financial crisis in 1997 and global financial crisis in 2007 and in this period China's overall export shows no dramatic variation.

Figure 4 illustrates that total spectrum of exporting countries per year is 64,108,113,135,149,158 and 170 respectively in the period of 2000 to 2006. The spectrum of 2006 is almost triplication of 2000 and there is very large rise between 2000 and 2001.

3 Both revenue- and quantity-based measures of productivity are monotonically related to the firm productivity drawn (Bernard, Redding and Schott, 2010.), and there is a positive correlation between them(Foster, Haltiwanger and Syverson, 29.1286 TD03s

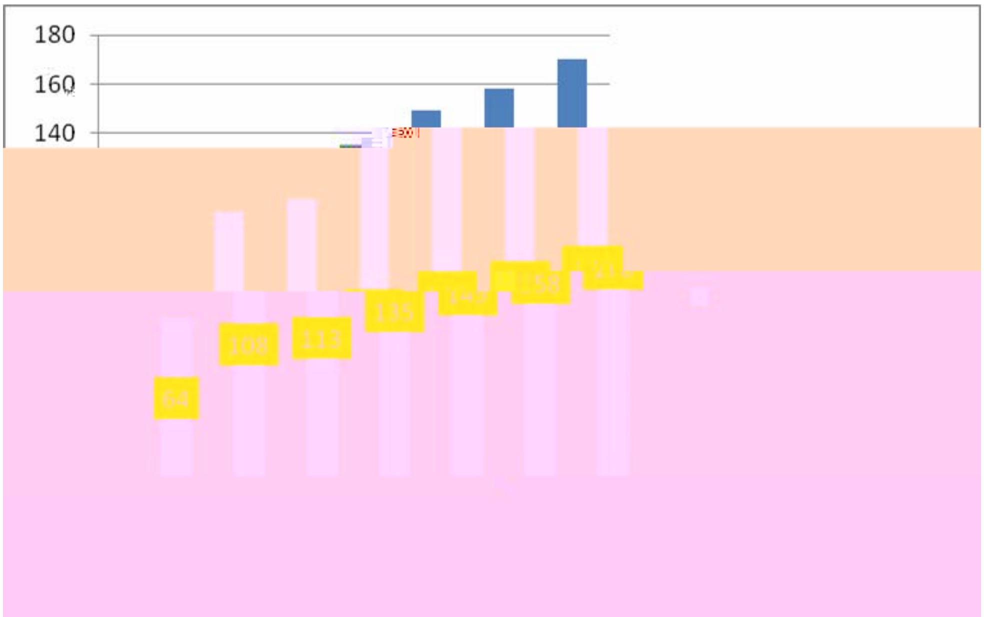

Figure 4: Total spectrum of exporting countries per year

Table 1 Descriptive statistics

Variables	Mean	SD	Min	Max
S	16.85	16.59571	1	83
R	25240.7	430875.8	3486	3641329
λ	499.5289	506.7539	11.97716	4422.953
$\bar{w}$	14.48639	8.23511	2.66187	54.955

Data source: National Bureau of Statistics of China and General Administration of Customs of the People's Republic of China.

In Table 1, the units of total revenue R, productivity λ, average wage $\bar{w}$ and spectrum of exporting countries S of a firm are thousand RMB, thousand RMB per labor, thousand RMB per person and number of export countries respectively. Here total revenue is the sum of sales revenue of a firm and its subsidy received from the government. Average wage is total wage which equals that actual wage plus welfare provided by the firm divided by the number of staff.

As to panel data estimations, the Hausman test (see the note of Table 2) for the difference between the Fixed-effects and Random-effects is not significant, suggesting that a Random-effect model is preferred over Fixed-effect model, so Random-effect method is employed.

Table 2　General estimates

autocorrelation: $F(1,51)$ 13.868, p 0.00; White's heteroskedasticity test: $F^2(9)=21.04$, $p<0.05$. t statistics in parentheses, 8 8 8 and 8 8 significant at 1% and 5% level respectively.

Furthermore, Wooldridge test for autocorrelation in panel data and modified Wald test for heteroskedasticity (see the note of Table 2) show that there is a significant first-order autocorrelation and heteroskedasticity within enterprises for the panel dataset.

In Table 2, these results of all three estimations show that: these three coefficients are all positive, s

Then

$$\frac{\partial n_E}{\partial \bar{w}} ? 0, \qquad \frac{\partial n_E}{\partial \theta} ? 0, \qquad \frac{\partial n_E}{\partial f_e} ? 0,$$

$$\frac{\partial n_E}{\partial S} ? 0, \quad \text{if } \theta \left(\frac{w - f_e}{f_e} \right)^{\frac{1}{\sigma}},$$

$$\frac{\partial n_E}{\partial S} ? 0, \quad \theta \left(\frac{w - f_e}{f_e} \right)^{\frac{1}{\sigma}}.$$

Assume that the marginal technology, the first variety's entry cost and the transaction cost are all constants. We can get the empirical model

$$n_{Eit} = \beta_0 + \beta_1 R_{it} + \beta_2 \theta_{it} + \beta_3 \bar{w}_{it} + \beta_4 S_{it} + u_i + v_{it},$$

Where θ_{it} is measured in terms of either revenue-based labor productivity or TFPR; company is denoted by i and t means year; u_i captures the company fixed effects and v_{it} is a stochastic error.

Table 3 Descriptive statistics

Variables	Mean	SD	Min	Max
R	5779290	1.09e+07	0	5.89e+07
θ	1067.232	1437.54	0	10291.55
$\bar{w}$	32.4892	26.01499	0	169.2304
S	11.21818	14.07041	0	66
n	4.036364	5.138201	0	38

Data source: Data source: National Bureau of Statistics of China and General Administration of Customs of the People's Republic of China.

The units of R, N, $\overline{w}$, S and n are thousand RMB, thousand RMB per labor, thousand RMB per person, number of export countries and number of heterogeneous varieties respectively. The value of zero means that the firm still isn't in business for that year.

Through Wooldridge test, autocorrelation exists in the panel data. With the result of Hausman test, the fixed method is be used here.

Table 4 General estimates

Variables	Fixed effect method	
R	1.59e-07***	(3.45)
N	0.0001913	(0.39)
$\overline{w}$	-.0103878	(-0.47)
S	0.2266864***	(10.56)
INTERCEPT	0.7061579	(1.09)
R-square	0.6471	
Observations	165	

Results are based on data of 33 automobile companies for 5 year period from 2001 to 2006 in China. The diagnostics tests results include: Hausman test F^{2} 3 $=12.99$, $p< 0.01$; f(p)Tm0 8 211534 0 TD0 Tc0 Tw

Figure 5 The interpretation of productivity's significance

Another is that: for Chinese automobile industry, the development of export variety is very slowly, the vast majority of its varieties are produced for the domestic market. From these two explanations we can see that the effect of productivity is also not significant.

6. Concluding Remarks

We adopt Melitz model (2003) and obtain the close-form solution of numbers of variety of differentiated goods and exporting markets, respectively. That is, one could know the number of exporting destination is fixed as long as the MNC achieves its maximum of its profit. Moreover, one also knows the trend of change of such number as the productivity grows. Furthermore, the number of differentiated goods could be determined by the firm simultaneously as the number of exporting markets as we shows above. The empirical test also confirms all of our results. It should extend current understanding for the performance of MNCs

On the other hand, this paper shows the number is a constant in equilibrium and then gives the upper bound of this number which is constant, moreover, we claims that the number of total varieties converge to this upper bound with productivity, but cannot reach this bound all the time.

The paper also proves that the function of variety with respect to productivity is concave.

For the exporting firms, we get the conclusion: there is a value which is determined by skilled labor’s wage, first variety’s entry cost and elasticity of substitution between any two goods., if the transaction cost is lower than it, the number of varieties increase with exporting countries; if higher than it, number would decrease.

At last, through empirical test, we know the signs of these four coefficients of revenue, productivity, average wage and number of exporting varieties are consistent with theoretical expectations. The firm’s revenue and number of exporting countries have significant effects on the change of export variety, however, the effects of productivity and average wage are not significant, then the paper provides reasonable explanations of these empirical results with the discussion of theoretical part and the fact of Chinese automobile industry.

Appendices

Appendix A:

Maximization of profit

$$\max_{S,M} \sum_{i=1}^{n} 3_{M/3}^{M} Mu \quad ª¬ \; 1 \quad SM^{1 \;\; V}{}_{i} \; º¼ v \quad ª¬ d \, W \; wn^{m} \quad n^{m} f_{e} \;\; S \;\; n^{m} f_{e} \quad F \; , M \qquad º¼ M$$

First order conditions:

Given S :

$$\sum_{i=1}^{n} Mu \qquad 1 \quad SM^{1 \;\; V}{}_{i} \qquad v \quad W m \;\; w \quad 1{+}s \; S \;\; f_{e} ª¬ n^{m \;\; 1} M c \qquad 0 , \quad M \qquad º¼$$

$$n^{m \;\; 1} n \; c \; M \quad \frac{MR \quad 1^{\sim} \; S \; W^{1 V}}{m \;\; w \quad 1{+}s \quad f_{e} ª¬ M W} \frac{M^{1} e^{V}}{3_{M/3}^{f} \; M^{1} \; e^{V} \; d \; M^{1} V} , \quad {}^{J} \quad {}^{M} \; º¼$$

Where $u \quad M v \quad M$ is equal to $J^{J \wedge}$.

—

$$n^m \quad \frac{MR \quad 1^{\sim} \; S \; W^{V}}{M \quad w^{\sim} \quad 1{+}s V \quad f_e^{a}} \frac{3^{M}_{M} \quad {}^1 e^{V} \, dM}{3^{f}_{M} \quad {}^1 e^{V} \, dM} \quad mb \,. \quad \frac{J \quad M \quad M}{W \quad M \quad o \tfrac{1}{4} \quad M}$$

And $n \quad M_D \quad 1$, then we yield

$$1 \quad n \quad M_D \, a^{m} \quad n^m \quad \frac{MR \quad 1^{\sim} \; S \; W^{V}}{M \quad o \tfrac{1}{4} \; w^{\sim} \quad 1{+}s V \quad f_e^{a}} \frac{3^{M}_{M} \quad {}^1 e^{V} \, dM}{3^{f}_{M} \quad {}^1 e^{V} \, dM} \quad mb \,, \quad \frac{J \quad M \quad M}{W \quad M \quad o \tfrac{1}{4} \quad M}$$

We get

$$mb \ldots$$

$$\frac{MR\,\tilde{A}(N^{*})}{BM\,V\,W f_e}\left\{ W^{\gamma}\frac{w - f_e}{f_e} - 1 \left[\frac{1}{W}\frac{w - f_e}{f_e} \right] - 0, \right\} !$$

Moreover, R and $A(N^{*})$ are positive, then we obtain

$$W^{\gamma}\frac{w - f_e}{f_e} - 1 \; ! \; 0,$$

That is

$$\max_{M} \sum_{i=1}^{n} \int_{M}^{M} Mu \;\; ¬M_i \;\; v \;\; S \;\; d ¼ \; Mn^m \; ¬n^m M \;\; f_e \;\; F, \quad (M) \; ¼$$

First order condition

$$\sum_{i=1}^{n} Mu \;\; M_i \;\; S\, v \;\; M \;\; m \;\; M \;\; f_e \;\; n^{m\;1} n \;\; C \;\; 0, \quad M$$

$$n^{m\;1} n \;\; C\, M \;\; \frac{MR}{m \;\; w \;\; f_e \;\; M \; V} \; \frac{M\;{}^{1} e^{\,V}}{\int^{f}_{M} \; M \;\; e^{\,V} \; d \; M} \; , \quad {}^{J}_{J} \;\; {}^{M}_{M}$$

Where $u \;\; M v \;\; M$ is equal to $e^{\,J\wedge}$.

$$\frac{1}{m} n^m \;\; \frac{MR}{M m \; V \;\; w^{\sim} \;\; f_e} \; \frac{\int_{M}^{M} \;\; {}^{1} e^{\,V} \; dM \;\; {}^{J} \;\; {}^{M} \;\; M}{\int^{f}_{M} \;\; {}^{1} e^{\,V} \; dM \;\; {}^{J} \;\; {}^{M} \;\; M} \;\; b,$$

Where b is constant.

$$n^m \;\; \frac{MR}{M \; V \;\; w^{\sim} \;\; f_e} \; \frac{\int_{M}^{M} \;\; {}^{1} e^{\,V} \; dM \;\; {}^{J} \;\; {}^{M} \;\; M}{\int^{f}_{M} \;\; {}^{1} e^{\,V} \; dM \;\; {}^{J} \;\; {}^{M} \;\; M} \;\; mb.$$

And $n \;\; M_D \;\; 1$, then we yield

$$1 \;\; n \;\; M_D \; ¬^{m} \;\; n^m \;\; \frac{MR \; ¼}{M \; V \;\; w^{\sim} \;\; f_e} \; \frac{\int_{M}^{M} \;\; {}^{1} e^{\,V} \; dM \;\; {}^{J} \;\; {}^{M} \;\; M}{\int^{f}_{M} \;\; {}^{1} e^{\,V} \; dM \;\; {}^{J} \;\; {}^{M} \;\; M} \;\; mb,$$

We get

$$mb \;\; 1,$$

So

$$\frac{\qquad}{\qquad} \; \frac{1}{1} \;\; 1^{\frac{1}{\;}}$$

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